AULA 04
INTRODUÇÃO AOS MÉTODOS ESPECTRAIS
PTC 5725 (09/10/2025)

Osvaldo Guimarães

Cheby Poly (I) T_n

$$\cos a \cdot \cos b = \left[\cos(a-b) + \cos(a+b)\right]/2$$

$$\cos(m\theta)\cdot\cos(n\theta) = \left[\cos((m-n)\theta) + \cos((m+n)\theta)\right]/2$$

$$\text{Se} \quad m = n = 0 \Rightarrow \int_0^\pi 1 \cdot d\theta = \pi.$$

$$\text{Se} \quad m = n \neq 0 \Rightarrow \int_0^\pi 1 \cdot d\theta/2 + \underbrace{\int_0^\pi \cos(k\theta)\cdot d\theta/2}_{0} = \pi/2 \quad k \in \mathbb{N}^*$$

$$\text{Se} \quad m \neq n \quad \Rightarrow \underbrace{\int_0^\pi \cos(k_1\theta)\cdot d\theta/2}_{0} + \underbrace{\int_0^\pi \cos(k_2\theta)\cdot d\theta/2}_{0} = 0$$

$$\boxed{\langle T_n | T_m \rangle_{w,\infty} = \begin{cases} 0 \quad \text{se } m \neq n \\ \pi/2, \text{ se } \ m = n \neq 0 \\ \pi \quad \text{ se } \ m = n = 0 \end{cases}}$$

Ortogonalidade

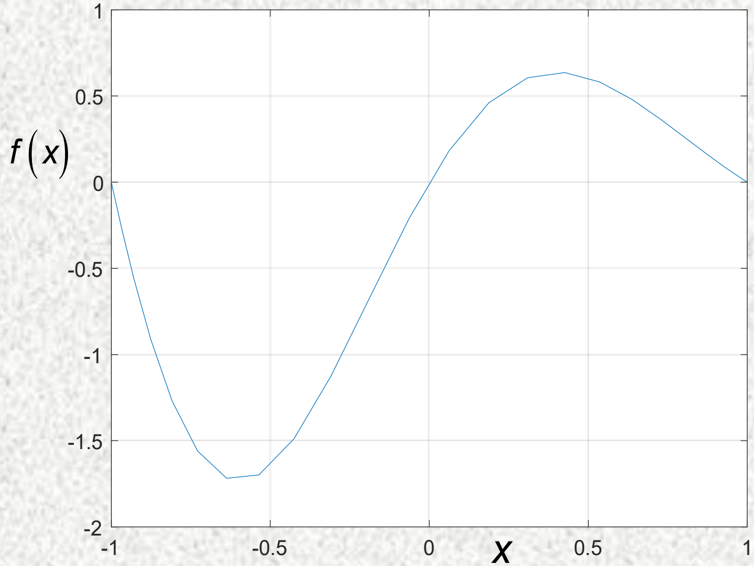
Seja $f(x) = c_0 T_0 + c_1 T_1 + \ldots + c_n T_n$, então

$$\int_{-1}^{1} f\, T_k w_{(x)}\, dx = \langle c_k T_k | T_k \rangle_{w,\infty} \quad \text{e como } \langle T_k | T_k \rangle_{w,\infty} = \pi/2$$

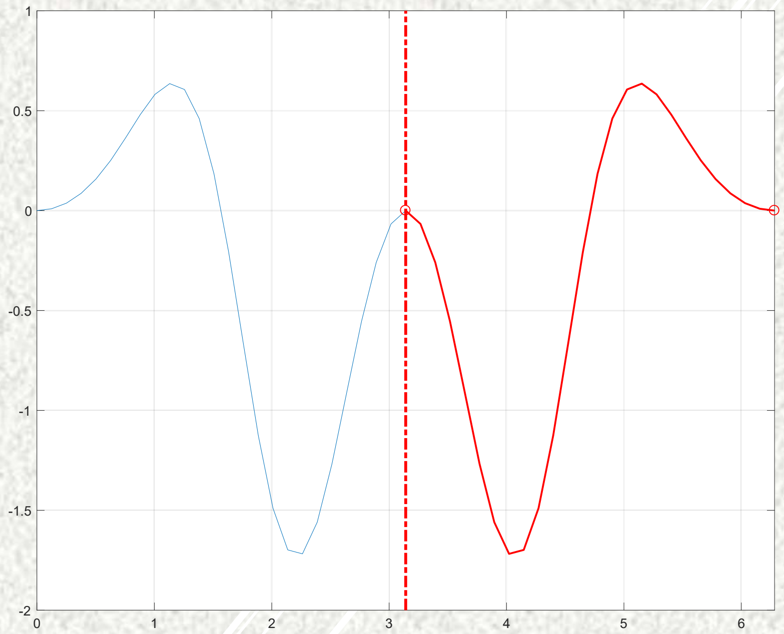
$$c_k = \frac{2}{\pi} \int_{-1}^{1} f\, T_k w_{(x)}\, dx$$

Quadratura – Discretização Cheby-Lobatto

Pontos: (n+1)+(n-1) = 2n



$$\Delta\theta = \frac{\pi}{n}$$



$$\langle T_n | T_n \rangle_{w,n} = \pi$$

Prova

$$g = g(\theta) = \cos^2(n\theta_j) = \cos^2\left(j \cdot n\frac{\pi}{n}\right) = 1$$

$$\langle T_n | T_n \rangle_{w,n} = \frac{1}{2}\Delta\theta \cdot \left[g_0 + 2(g_1 + g_2 \ldots + g_{n-1}) + g_n\right]$$

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Prova

$$\langle T_n | T_n \rangle_{w,n} = \pi$$

$$\theta_j = j \cdot \Delta\theta = j\frac{\pi}{n}$$

$$g = g(\theta) = \cos^2(n\theta_j) = \cos^2\left(j \cdot n\frac{\pi}{n}\right) = 1$$

$$\langle T_n | T_n \rangle_{w,n} = \frac{\pi}{2n} \cdot \left[1 + 2n - 2 + 1\right] = \pi$$

$$\int_{-1}^{1}\frac{f\cdot T_k dx}{\sqrt{1-x^2}}=\int_0^{\pi}G(\theta)\,d\theta=\frac{1}{2}\int_0^{2\pi}G\cdot d\theta$$

$$=\frac{1}{2}\sum_{k=0}^{2n-1}{}''G_k\cdot\Delta\theta \qquad \text{N.B. } G_k=f\left(x_j\right)\cdot T_k\left(x_j\right)$$

$\Delta\theta=\dfrac{\pi}{n},\ G_0$ e G_n só aparecem uma vez no somatório.

$$c_k=\frac{2}{\pi}\cdot\left(\frac{1}{2}\right)\left(\frac{\pi}{n}\right)\left[G_0+2\left(G_1+..+G_{n-1}\right)+G_n\right] \qquad 0\neq k\neq n$$

$$c_k=\frac{1}{n}\left[G_0+2\left(G_1+..+G_{n-1}\right)+G_n\right] \quad 0\neq k\neq n$$

$$c_n=\frac{1}{2n}\left[G_0+2\left(G_1+..+G_{n-1}\right)+G_n\right] \qquad k=n$$

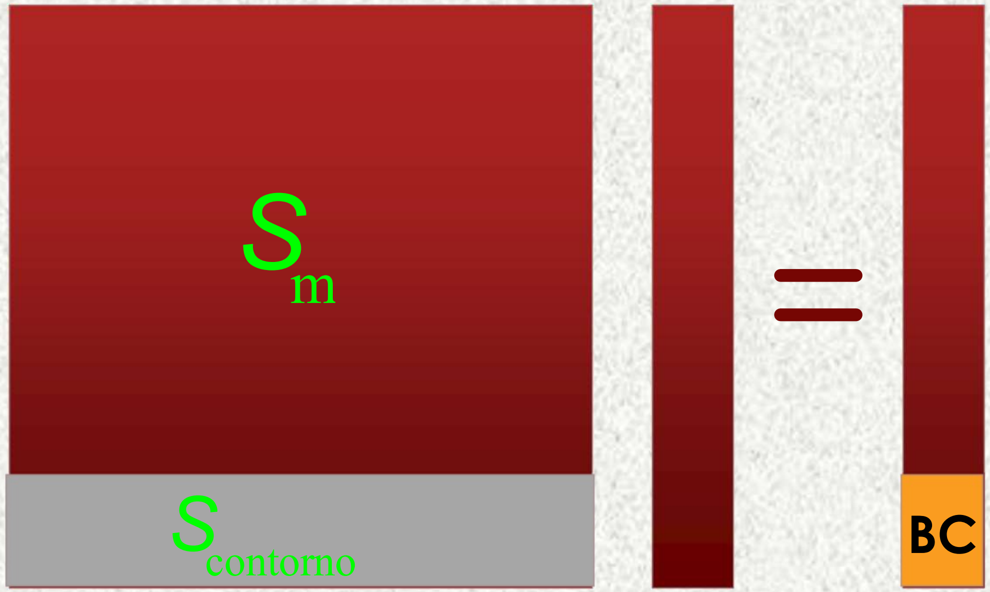
$$c_0=\frac{1}{2n}\left[G_0+2\left(G_1+..+G_{n-1}\right)+G_n\right] \qquad k=0$$

Matriz que determina os coeficientes da expansão.

```matlab
% Bases of Chebyshev
% output [B,invB] Basis and its inverse]
% input: n polynomial order

function [B,invB,xL] = Base_Cheby_Lobatto(n)
    m = floor(n/2);
        if rem(n,2) == 1
            xm = -cos( (0:m).'*pi/n);
            xL = [xm;-flipud(xm)];
        else
            xm = -cos( (0:m-1).'*pi/n);
            xL = [xm;0;-flipud(xm)];
        end
            B = ones(n+1);  t = (n:-1:0).'*pi;
    for k = 1:n
        B(:,k+1) = cos(k*t/n);
    end
%% inverse matrix
    invB = B.'/n;
    invB(:,2:n) = 2*invB(:,2:n);
    invB([1,n+1],:) = invB([1,n+1],:)/2;
end
```

C:\Users\osval\Dropbox\CURSO POLI 23\2025\Rotinas\Solver2nd.m

```
%% Solving 2nd order ODE   - Solution: u(x) = x.^5-2*x+1;
function un = Solver2nd(n)

xs = -cos((0:n)'*pi/n);   f = @(x) x.^5-2*x+1;
D = Generalized_Diff_Mat(xs); D2 = D^2;
II = eye(n+1); Ibb = II(2:n,:);
%% System withou BC
Sm = Ibb*( diag(xs.^2/20)*D2 + diag(xs)*D - II ); rm = Ibb*(5*xs.^5-1);
Sc = zeros(2,n+1);  Sc(1,1)= 1; Sc(2,n+1) = 1;  rc = [2;0];

%% System
S = [Sm;Sc];   RHS = [rm;rc];
un = S\RHS;

```



$$S \cdot \vec{y} = \vec{r}$$

Cheby Poly (I) T_n

$$x \in [-1,1] \qquad T_n(\cos\theta) = \cos(n\theta) \qquad x = \cos\theta$$

$$\theta \in [0,\pi] \qquad T_0(x) = 1 \text{ e } \quad T_1(x) = x \qquad \frac{dx}{d\theta} = -\sin\theta$$

$$T_n(\cos\theta) = \cos(n\theta) = g_n(\theta)$$

$$I_{m,n} = \int_{-1}^{1} T_n \cdot T_m \cdot \frac{1}{\sqrt{1-x^2}}\, dx = \int_{\pi}^{0} g_n(\theta) \cdot g_m(\theta) \frac{1}{\sin\theta}(-\sin\theta)\, d\theta$$

$$I_{m,n} = \int_{0}^{\pi} g_n(\theta) \cdot g_m(\theta)\, d\theta$$

Base de Chebyshev Tipo I

$$f(x) \cong \sum_{k=0}^{n} c_k T_k(x) = \begin{bmatrix} T_0 & T_1 \ldots T_n \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = \langle B_T | C \rangle$$

Análogo:

$$\vec{v} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = 3\hat{i} + 2\hat{j} + 4\hat{k}$$

Recurrence Formulas for $T_n(x)$

When the first two Chebyshev polynomials $T_0(x)$ and $T_1(x)$ are known, all other polynomials $T_n(x)$, $n \geq 2$ can be obtained by means of the recurrence formula

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

$$2T_m(x)T_n(x) = T_{m+n}(x) + T_{|m-n|}(x)$$

which is an analogy to the addition theorem

$$2\cos\alpha\,\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$\int T_n(x)dx = \frac{1}{2}\left[\frac{T_{n+1}(x)}{(n+1)} - \frac{T_{n-1}(x)}{(n-1)}\right] + C \quad n \geq 2$$

Osvaldo Guimarães

Método de Horner

Consider the polynomial

$$p_6(x) = (x+8)(x+5)(x+3)(x-2)(x-3)(x-7)$$

```
>> syms x;
>> P = [1 4 -72 -214 1127 1602 -5040];
>> f(x) = poly2sym(P)

f(x) =

x^6 + 4*x^5 - 72*x^4 - 214*x^3 + 1127*x^2 + 1602*x - 5040

>> h = horner(f)

h(x) =

x*(x*(x*(x*(x*(x + 4) - 72) - 214) + 1127) + 1602) - 5040
```

$$
\begin{aligned}
T_0(x) &= 1\\
T_1(x) &= x\\
T_2(x) &= 2x^2-1\\
T_3(x) &= 4x^3-3x\\
T_4(x) &= 8x^4-8x^2+1\\
T_5(x) &= 16x^5-20x^3+5x\\
T_6(x) &= 32x^6-48x^4+18x^2-1\\
T_7(x) &= 64x^7-112x^5+56x^3-7x\\
T_8(x) &= 128x^8-256x^6+160x^4-32x^2+1\\
T_9(x) &= 256x^9-576x^7+432x^5-120x^3+9x\\
T_{10}(x) &= 512x^{10}-1280x^8+1120x^6-400x^4+50x^2-1\\
T_{11}(x) &= 1024x^{11}-2816x^9+2816x^7-1232x^5+220x^3-11x
\end{aligned}
$$

$$
x^n = 2^{1-n}\sum_{k=0}^{\lfloor n/2\rfloor}\binom{n}{k}T_{n-2k}(x)
$$

$$
\begin{aligned}
1 &= T_0\\
x &= T_1\\
x^2 &= \frac{1}{2}(T_0+T_2)\\
x^3 &= \frac{1}{4}(3T_1+T_3)\\
x^4 &= \frac{1}{8}(3T_0+4T_2+T_4)\\
x^5 &= \frac{1}{16}(10T_1+5T_3+T_5)\\
x^6 &= \frac{1}{32}(10T_0+15T_2+6T_4+T_6)\\
x^7 &= \frac{1}{64}(35T_1+21T_3+7T_5+T_7)\\
x^8 &= \frac{1}{128}(35T_0+56T_2+28T_4+8T_6+T_8)\\
x^9 &= \frac{1}{256}(126T_1+84T_3+36T_5+9T_7+T_9)\\
x^{10} &= \frac{1}{512}(126T_0+210T_2+120T_4+45T_6+10T_8+T_{10})\\
x^{11} &= \frac{1}{1024}(462T_1+330T_3+165T_5+55T_7+11T_9+T_{11})
\end{aligned}
$$

Special Values of $T_n(x)$

The following special values and properties of $T_n(x)$ are often useful:

$$T_n(-x) = (-1)^n T_n(x) \qquad T_{2n}(0) = (-1)^n$$

$$T_n(1) = 1 \qquad T_{2n+1}(0) = 0$$

$$T_n(-1) = (-1)^n$$

$$D_{i+1,j+1} = \begin{cases} j \text{ if } i = 0 \text{ and } j \text{ odd} \\ 2j, \text{ if } 0 < i < j, \quad i + j \text{ odd.} \end{cases}$$

Roots: Chebyshev-Gauss points

$$x_i = \cos\left(\frac{2i-1}{2N}\pi\right) \quad i = 1 : N \qquad \text{ou}$$

$$x_j = \cos\left(\frac{j+1/2}{n+1}\pi\right) \quad j = 0 : n$$

- *For Chebyshev-Gauss-Lobatto* (CGL) *quadrature,*

$$x_j = -\cos\frac{\pi j}{N}, \quad \omega_j = \frac{\pi}{\tilde{c}_j N}, \quad 0 \le j \le N.$$

where $\tilde{c}_0 = \tilde{c}_N = 2$ *and* $\tilde{c}_j = 1$ *for* $j = 1, 2, \ldots, N-1$.

With the above choices, there holds

$$\int_{-1}^{1} p(x)\frac{1}{\sqrt{1-x^2}}dx = \sum_{j=0}^{N} p(x_j)\omega_j, \quad \forall p \in P_{2N+\delta},$$

where $\delta = 1, 0, -1$ *for the* CG, CGR *and* CGL, *respectively.*

In the Chebyshev case, the nodes $\{\theta_j = \arccos(x_j)\}$ are equally distributed on $[0,\pi]$, whereas $\{x_j\}$ are clustered in the neighborhood of $x = \pm 1$ with density $O(N^{-2})$, for instance, for the CGL points

$$1 - x_1 = 1 - \cos\frac{\pi}{N} = 2\sin^2\frac{\pi}{2N} \simeq \frac{\pi^2}{2N^2} \quad \text{for } N \gg 1.$$

J. Shen, T. Tang, and L.-L. Wang, Spectral Methods: Algorithms, Analysis and Applications, vol. 41. p. 108 Berlin, Heidelberg: Springer, 2011.

Produto de Hadamard

For two matrices A and B of the same dimension $m \times n$, the Hadamard product $A \circ B$ (or $A \odot B$[1][5][6][7]) is a matrix of the same dimension as the operands, with elements given by

$$(A \circ B)_{ij} = (A)_{ij}(B)_{ij}.$$

Example

For example, the Hadamard product for a 3 × 3 matrix A with a 3 × 3 matrix B is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \circ \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}\, b_{11} & a_{12}\, b_{12} & a_{13}\, b_{13} \\ a_{21}\, b_{21} & a_{22}\, b_{22} & a_{23}\, b_{23} \\ a_{31}\, b_{31} & a_{32}\, b_{32} & a_{33}\, b_{33} \end{bmatrix}.$$

Propriedades

$$\begin{aligned} A \circ B &= B \circ A, \\ A \circ (B \circ C) &= (A \circ B) \circ C, \\ A \circ (B + C) &= A \circ B + A \circ C, \\ (kA) \circ B = A \circ (kB) &= k\,(A \circ B)\,, \\ A \circ 0 = 0 \circ A &= 0. \end{aligned}$$

EDO com coeficientes varíaveis

Seja a EDO: $\alpha_2 \cdot y_{xx} + \alpha_1 \cdot y_x + \alpha_0 \cdot y = e,$ c/ $x \in \left[-1,1\right] \subset \mathbb{R}$

na qual todos os termos são dependentes de x.

Discretizada matricialmente para a abordagem pseudo-espectral, a equação fica:

$\alpha_2 \circ \left(D^2 \cdot y\right) + \alpha_1 \circ \left(D \cdot y\right) + \alpha_0 \circ y = e,$

onde o símbolo "$\circ$" siginifica o produto de Hadamard (termo a termo).

Vamos considerar a parcela $\alpha_2 \circ \underbrace{D^2 \cdot y}_{Z}$, observando que α_2 e Z são vetores coluna, pondo

$\alpha_2 = \alpha_{n,1}$ e $Z = Z_{n,1}$, então $\quad \alpha \circ Z = C$, tal que $c_i = \alpha_i \cdot Z_i, = \; i = 1:n.$

Considerando a matriz diagonal $\alpha_{n,n}$, tal que os termos não nulos são $\alpha_{i,i} = \alpha_i$

e o produto matricial convencional $\alpha \cdot Z = D$, teremos:

$d_i = \alpha_{i,i} \cdot Z_i$, portanto: $\alpha \circ Z = \mathrm{diag}\left(\alpha\right) \cdot Z.$

Exemplo

$\alpha = \left[1,2,3,4\right]^T$ e $Z = \left[5,2,3,1\right]^T$, então:

$$\alpha \circ Z = \begin{bmatrix} 5 \\ 4 \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & \cdots & & 0 \\ \vdots & 2 & & \\ & & 3 & \\ 0 & & & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \mathrm{diag}\left(\alpha\right) \cdot Z$$

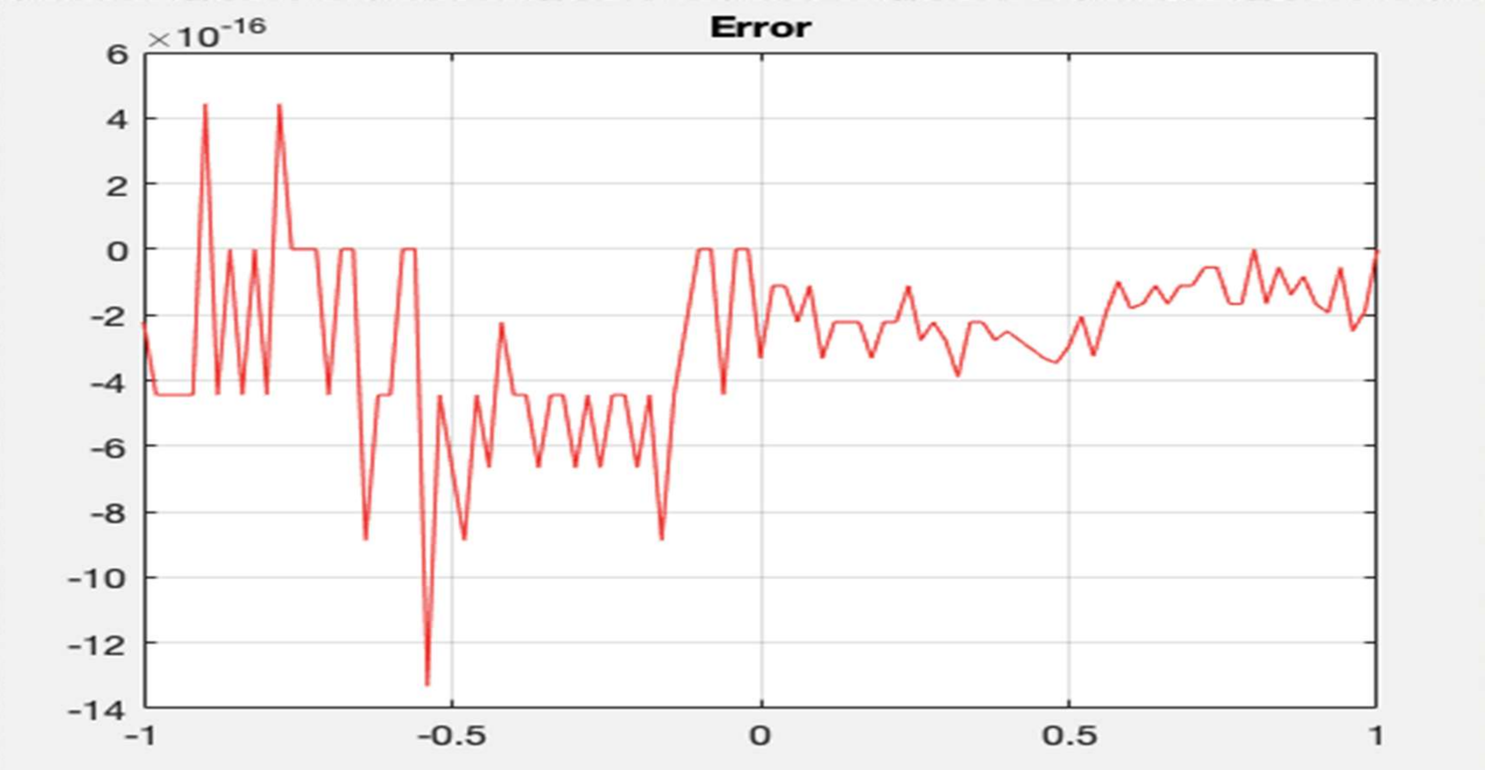
Seja a EDO:

$$\frac{x^2}{20}y'' + x \cdot y' - y - 5x^5 + 1 = 0,$$

com $y(-1) = 2$ e $y(1) = 0$

Resolver espectralmente com expansão de ordem 7.

Solução analítica: $y = x^5 - 2x + 1$



Integração e diferenciação no espaço espectral

$$Q(u) = a_0 + a_1 u + \ldots + a_n u^n + 0 \cdot u^{n+1}$$

$$\frac{d}{dx}\begin{bmatrix} x^0 \\ x \\ .. \\ x^n \end{bmatrix} = Z \cdot \begin{bmatrix} x^0 \\ x \\ .. \\ x^n \end{bmatrix}, \text{ com } \begin{cases} \frac{d}{dx}[x^0] = 0, \\ \ldots, \\ \frac{d}{dx}[x^n] = n \cdot x^{n-1} \end{cases}$$

$$f(x) = \langle B | C \rangle = \langle C^T | B^T \rangle$$

$$f'(x) = \langle C^T | Z | B^T \rangle = \langle B | Z^T | C \rangle,$$

$$\text{logo } D_S = Z^T$$

$$\text{então: } Z_{i,i-1} = i - 1 \qquad i = 2 : n-1$$

$$Z = \begin{pmatrix} 0 & & & & \\ 1 & 0 & & & \\ & 2 & & & \\ & & 3 & & \\ & & & \vdots & \end{pmatrix}$$

Teorema do Sandwich

Todas as matrizes operacionais polinomiais, ortogonais ou não, perfazem uma classe de similaridade.

$$D_G = B_G \Omega (B_G)^{-1} \qquad \Omega = Z^T$$

Há expressões fechadas para várias bases polinomiais.

$$D_{\text{Cheby}} = \begin{bmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exemplo elementar

$y'(x) = 4x \quad y(1) = 1 \quad x \in [-1,1]$

$D\hat{y} = 4T_1 \quad$ e $\quad [1\ 1\ 1\ 1\ 1] \cdot \hat{y} = 1$

$$\boxed{D_{i+1,j+1} = \begin{cases} j \text{ if } i = 0 \text{ and } j \text{ odd} \\ 2j, \text{ if } 0 < i < j, \quad i + j \text{ odd.} \end{cases}}$$

$$\begin{bmatrix} 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \hat{y} = \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{y} = [0\ 0\ 1\ 0\ 0]^T \Rightarrow y(x) = T_2(x) = 2x^2 - 1$$

Método de Newton

Equação transcendental

$$\text{sen}\left(\frac{2{,}002}{R}\right) = \frac{2}{R} \Rightarrow R = 25{,}854752670654$$

Coeficiente angular da tangente à curva

$$f'(x_0) = \frac{f(x_0) - 0}{x_0 - x_1}$$

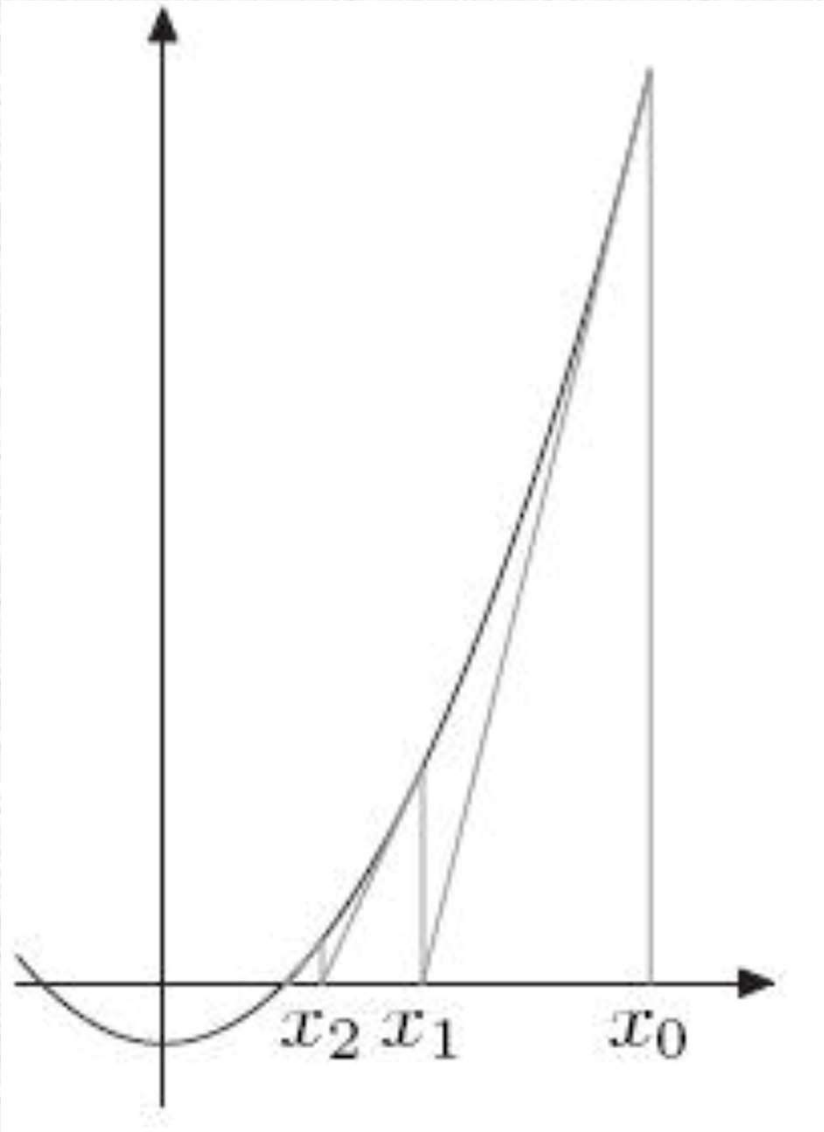
Prosseguindo: $x_0 - x_1 = \dfrac{f(x_0)}{f'(x_0)}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Agora, x_1 é o novo candidato e: $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}.$

Genericamente

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$



Sistema de ecuaciones no lineales

$$\begin{pmatrix} x'_1 \\ x'_2 \\ \dots \\ x'_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} - \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}^{-1} \begin{pmatrix} f_1(x_1, x_2 \dots x_n) \\ f_2(x_1, x_2 \dots x_n) \\ \dots \\ f_n(x_1, x_2 \dots x_n) \end{pmatrix}$$

Prof. Angel Garcia - Anexo

Ejercicios

Resolver el sistema de ecuaciones no lineales

$$\begin{cases} x^3 + y = 1 \\ y^3 - x = -1 \end{cases}$$

Comprobando que su solución es (1,0)

Resolver el sistema de ecuaciones no lineales

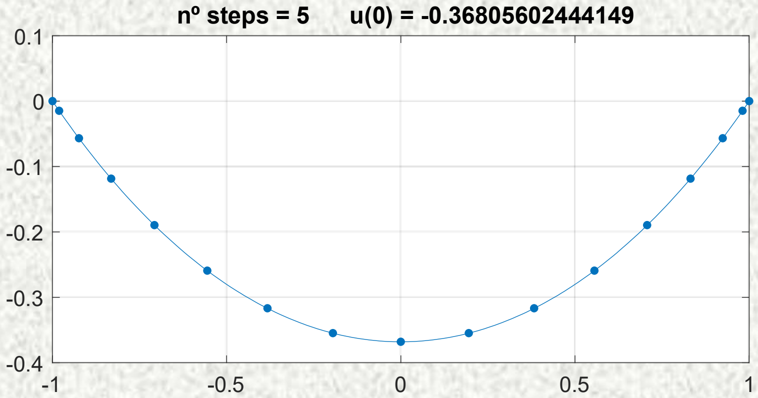
$$\begin{cases} \sin(xy) + \exp(-xz) - 0.9 = 0 \\ z\sqrt{x^2 + y^2} - 6.7 = 0 \\ \tan\left(\frac{y}{x}\right) + \cos z + 3.2 = 0 \end{cases}$$

Tomando x_0=1, y_0=2 y z_0=2 como aproximación inicial

Código p/ solução

$$y'' = e^{y} \qquad y(\pm 1) = 0 \quad x \in [-1,1]$$

com o Jacobiano

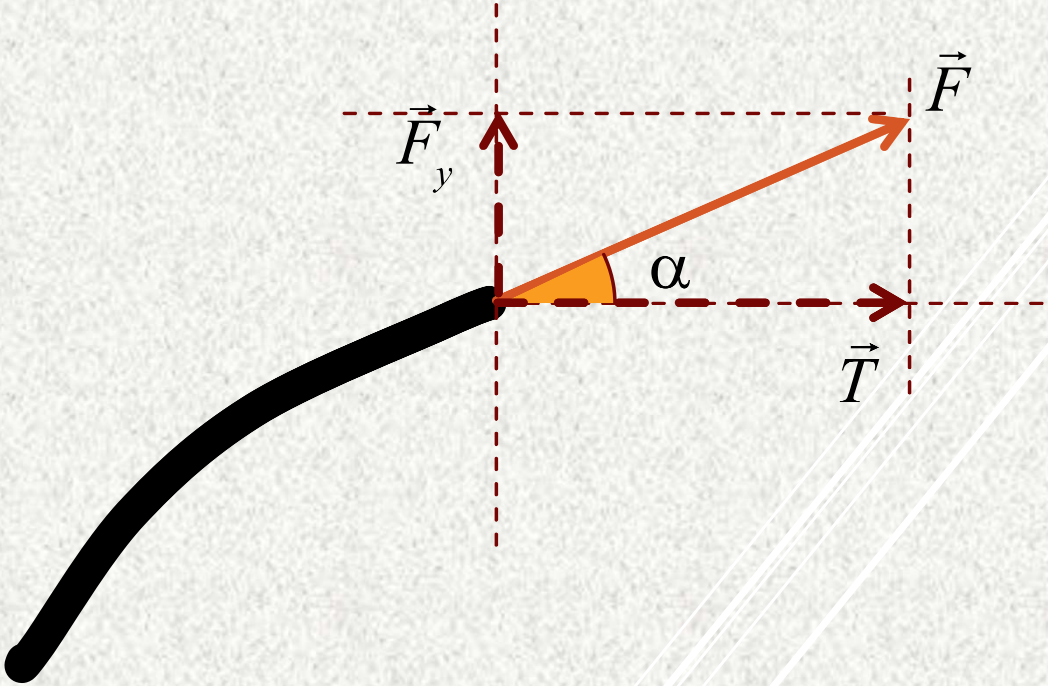



```
% Example of Non-Linear 2nd order ODE

function u = Ullexpu(n)

[~,~,xL] = Base_Cheby_Lobatto(n);
D = Generalized_Diff_Mat(xL); D2 = D^2;
II = eye(n+1); Ibb = II(2:n,:);
J = zeros(n+1); J(n,1) = 1; J(n+1,n+1) = 1;
bc = [0;0];
%% Guess
u = zeros(n+1,1); u = (xL.^2-1)/3;
%% Loop - Newton's method
for k = 1:50
    J(1:n-1,:) = Ibb*( D2 - diag(exp(u)) );
    % System for a given u
    r = [Ibb*(D2*u - exp(u));bc-[u(1);u(n+1)]];

        du = -J\r; k

    u = u+du;
    if norm(du)<1e-13
        break
    end
end

%% Plots
xp = -1+ 2*(0:100).'/100; up = Bary_Generic(xL,u,xp);
plot(xL,u,'ro','MarkerSize',8);
hold on
plot(xp,up,'LineWidth',2);
grid on

end
```

Condições de contorno

$$\frac{\partial U}{\partial x} = \tan\alpha \quad F_y = T \cdot \tan\alpha$$

$$\mathscr{P}_{\text{otência}} = F \cdot v = F_y \cdot \dot{U}$$



$$\dot{U} = 0 \Rightarrow U\left(x_b, t\right) = \text{constante} \quad \left(\text{Dirichlet}\right)$$

$$\mathscr{P}_{\text{otência}} = 0, \text{ mas o extremo não é fixo } \Rightarrow \frac{\partial U\left(x_b, t\right)}{\partial x} = 0 \left(\text{Neumann}\right)$$

TAREFA

. By means of the recurrence formula obtain Chebyshev polynomials $T_2(x)$ and $T_3(x)$ given $T_0(x)$ and $T_1(x)$.

. Show that $T_n(1) = 1$ and $T_n(-1) = (-1)^n$

. Show that $T_n(0) = 0$ if n is odd and $(-1)^{n/2}$ if n is even.

. Setting $x = \cos\theta$ show that

$$T_n(x) = \frac{1}{2}\left[\left(x + i\sqrt{1-x^2}\right)^n + \left(x - i\sqrt{1-x^2}\right)^n\right]$$

where $i = \sqrt{-1}$.

. Find the general solution of Chebyshev's equation for $n = 0$.

. Obtain a series expansion for $f(x) = x^2$ in terms of Chebyshev polynomials $T_n(x)$.

$$x^2 = \sum_{n=0}^{3} A_n T_n(x)$$

. Express x^4 as a sum of Chebyshev polynomials of the first kind.

Resolva pelo método de Newton

$$y'' = e^y \qquad y(\pm 1) = 1 \quad x \in [-1,1]$$

Considerando $R(x) = y'' - e^y$, faça o gráfico $R \times x$.

Resolva os exercícios propostos do Prof. Angel Garcia.