Optimal Transport Networks in Spatial Equilibrium

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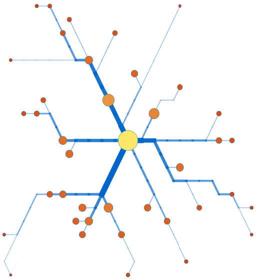

Introduction

- Every year the world economy invests lots of resources to develop transport infrastructre
  - 20% of World Bank spending, 6% of governments' budgets worldwide
- Growing literature trying to understand/quantify the role of transport infrastructure on economy
  - Transport costs are important determinants of trade costs (e.g., Limao and Venables, 2001)
  - Large impact on economic outcomes (Donaldson et al., 2016; Allen and Arkolakis, 2016)
- How should we design the transport network to maximize welfare?

This Paper

- **Develop a framework to study optimal transport networks in general equilibrium**

1. **Solve a global optimization over the space of networks**
   - given any primitive fundamentals
   - in a neoclassical trade framework (with labor mobility)
2. **Apply to actual road networks in 24 European countries**
   - how large are the gains from expansion and the losses from misallocation of current networks?
   - how to these effects vary across countries?
   - what are the regional effects?

Key Features

- **Neoclassical Trade Model on a Graph**
  - Infrastructure impacts shipping cost in each link
- **Sub-Problems:**
  - how to ship goods through the network? ("Optimal Flows")
  - how to build infrastructure? ("Optimal Network")
- **Optimal flows/routes = well known problem in Optimal Transport literature**
  - Numerically very tractable
  - Especially using dual approach = convex optimization in space of prices
- **Here: Full problem (Flows+Network+GE) inherits numerical tractability**
  - Infrastructure investment expressed as function of equilibrium prices
  - For sufficiency and tractability, add congestion in transport ($\rightarrow$global optimum)

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Literature Background

- Canonical quantitative trade frameworks: Eaton and Kortum (2002), Anderson and van Wincoop (2003)
- Counterfactuals with respect to infrastructure in gravity models: Allen and Arkolakis (2014), Redding (2016), Nagy (2016), Ahlfeldt, Redding and Sturm (2016), Sotelo (2016),...
- First-order welfare impact of changes in infrastructure: Allen and Arkolakis (2019)
- Optimization or search over networks: Felbermayr and Tarasov (2015), Alder (2019)
- Empirical assessment of actual changes in transport costs: Chandra et al. (2000), Baum-Snow (2007), Feyrer (2009), Donaldson (2012), Duranton et al. (2014), Pascali (2014), Faber (2014), Donaldson and Hornbeck (2016),...
- Here
  - Global optimization over networks in a spatial equilibrium
  - Related to optimal flow problem on a network (e.g., Chapter 8 of Galichon, 2016)
    - OT literature does not typically embed optimal-transport in GE or optimize over network

Plan

1. The framework
2. Illustrative Examples
3. Application

Preferences and Technologies

- $\mathcal{J} = \{1, \ldots, J\}$ locations
  - N traded goods aggregated into c_j
  - 1 non-traded good h_j in fixed supply (can make it variable)
  - L_j workers located in j (fixed or mobile)

- Homothetic and concave utility in j,

$$U\left(c_j, h_j\right)$$

  where

$$c_j L_j = C_j^T\left(C_j^1, \ldots, C_j^N\right)$$

  - $C_j^T(\cdot)$ homogeneous of degree 1 and concave (e.g., CES)

- Output of sector n in location j is:

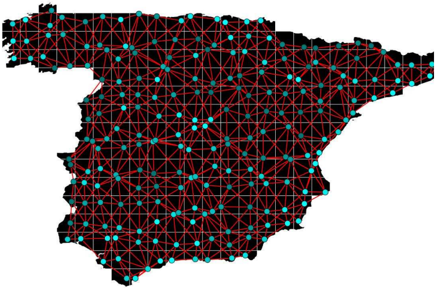
$$Y_j^n = F_j^n\left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\right)$$

  - $F_j^n(\cdot)$ is either neoclassical or a constant
  - $\mathbf{V}_j^n, \mathbf{X}_j^n$ = other primary factors and intermediate inputs

- Special cases
  - Ricardian model, Armington Specific-factors, Heckscher-Ohlin, Endowment economy, Rosen-Roback...

Underlying Graph

- The locations are arranged on an *undirected* graph
  - $\mathcal{J} = \{1, ..., J\}$ nodes
  - $\mathcal{E}$ edges
- Each location j has a set $\mathcal{N}(j)$ of "neighbors" (directly connected)
  - Shipments flow through neighbors
  - "Neighbors" may be geographically distant
    - ⋆ Fully connected case $\mathcal{N}(j) = \mathcal{J}$ is nested
- Example: Spain, $\sim$50 km$\times$50 km square network, $\#\mathcal{N}(j) = 8$



Transport Technology

- Q_{jk}^n = quantity of commodity n from j to $k \in \mathcal{N}(j)$
  - Per-unit cost $\tau_{jk}^n\left(Q_{jk}^n, I_{jk}\right)$ denominated in units of good itself (iceberg)
  - I_{jk} = index of road quality/capacity (number of lanes, highway,...)
- **Decreasing returns to transport:** $\frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} \geq 0$
  - "Congestion" in short, may account for travel times, road damage, fixed factors in transport technologies...
  - Alternatively, cross-good congestion: $\tau_{jk}^n\left(\sum_n m_n Q_{jk}^n, I_{jk}\right)$ denominated in units of the bundle of traded goods
- **Positive returns to infrastructure:** $\frac{\partial \tau_{jk}^n}{\partial I_{jk}} < 0$
  - Infrastructure investment lower trade costs (# of lanes, whether road is paved, etc.)
- **Building infrastructure I_{jk}** takes up $\delta_{jk}^I I_{jk}$ units of a scarce resource in fixed supply K ("asphalt")
  - δ_{jk}^I may vary across links due to ruggedness, distance...
  - The transport network is given by $\left\{I_{jk}\right\}_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$

Decentralized Allocation Given the Network

- **Free entry of atomistic traders into shipping each n from o to d for all $(o,d) \in \mathcal{J}^2$**
  - Problem of traders shipping from o to d

$$\min_{r=(j_0,\ldots,j_\rho)\in\mathcal{R}_{od}} \underbrace{p_o^n}_{\text{Sourcing Costs}} + \underbrace{\sum_{k=0}^{\rho-1} p_{j_k}^n \tau_{j_k j_{k+1}}^n}_{\text{Transport costs}} + \underbrace{\sum_{k=0}^{\rho-1} p_{j_k}^n t_{j_k j_{k+1}}^n}_{\text{Taxes}}$$

    - $\mathcal{R}_{od}$ are all possible routes from o to d
    - Corresponds to minimum-cost route problem from gravity literature in the absence of taxes

- **Resource constraint (conservation of flows)**

$$C_j^n + \sum_{n'} X_j^{nn'} + \underbrace{\sum_{k\in\mathcal{N}(j)} \left(1+\tau_{jk}^n\right) Q_{jk}^n}_{\text{Exports}} \le Y_j^n + \underbrace{\sum_{i\in\mathcal{N}(j)} Q_{ij}^n}_{\text{Imports}}$$

- **Remaining features correspond to standard decentralized competitive equilibrium (given I_{jk})**

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Planner's Problem Given Network (Labor Mobility)

Definition

The planner's problem given the infrastructure network is

$$W_0\left(\left\{I_{jk}\right\}\right)=\max_{\mathbf{C}_j,\mathbf{L}_j,\mathbf{V}_j^n,\mathbf{X}_j^n,L_j}\max_{Q_{jk}^n} u$$

subject to (i) availability of traded and non-traded goods,

$$c_jL_j \leq C_j^T\left(\mathbf{C}_j\right) \text{ and } h_jL_j \leq H_j \text{ for all } j;$$

(ii) the balanced-flows constraint,

$$C_j^n+\sum_{n'}X_j^{nn'}+\sum_{k\in\mathcal{N}(j)}\left(Q_{jk}^n+\tau_{jk}^n\left(Q_{jk}^n,I_{jk}\right)Q_{jk}^n\right)=F_j^n\left(L_j^n,\mathbf{V}_j^n,\mathbf{X}_j^n\right)+\sum_{i\in\mathcal{N}(j)}Q_{ij}^n \text{ for all } j,\ n;$$

(iii) free labor mobility,

$$L_ju \leq L_jU\left(c_j,h_j\right) \text{ for all } j;$$

(iv) local and aggregate labor-market clearing; and
(v) factor market clearing and non-negativity constraints.

◂ Immobile Labor

Proposition

(Decentralization) Given the network, the welfare theorems hold with Pigovian taxes on shipping companies.

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Immobile Labor

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Optimal Flows Problem

- No-arbitrage conditions for shipping companies

$$P_k^n \leq P_j^n \left(1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n\right), = \text{if } Q_{jk}^n > 0$$

- Dual solution coincides with primal ▸ detail
  - Dual = convex optimization with linear constraints in smaller space (just prices)
  - Efficient algorithms are guaranteed to converge to global optimum (Bertsekas, 1998)

Optimization over Transport Network

Definition

The full planner's problem with labor mobility is

$$W = \max_{\{I_{jk}\}} W_0\left(\{I_{jk}\}\right)$$

subject to:

(a) the network building constraint, $\sum_j \sum_{k\in\mathcal{N}(j)} \delta^I_{jk} I_{jk} = K$*; and*

(b) the bounds $\underline{I}_{jk} \leq I_{jk} \leq \overline{I}_{jk}$

- At the global optimum, the optimal network satisfies

$$\underbrace{\mu\delta^I_{jk}}_{\text{Building Cost}} \geq \underbrace{\sum_n P^n_j Q^n_{jk} \times \left(-\partial\tau^n_{jk}/\partial I_{jk}\right)}_{\text{Gain from Infrastructure}}, \; = \text{ if } I_{jk} > \underline{I}_{jk}$$

  - Reduces optimization to space of prices$\longrightarrow$ Full problem inherits tractability of optimal flows

Proposition

If the function $Q\tau_{jk}(Q, I)$ is convex in Q and I, the full planner's problem with mobile labor (immobile labor) is a quasiconvex (convex) optimization problem.

- Ensures that our solution is indeed a global optimum for the transport network
- E.g., i f$\tau_{jk} = \delta^\tau_{jk} Q^\beta I^{-\gamma}$, convexity holds if $\beta \geq \gamma$

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Additional Results

1. Tree property in non-convex cases ▸ Proposition
2. Inefficiencies and externalities in the market allocation ▸ Proposition
3. Computational aspects ▸ Computation

Plan

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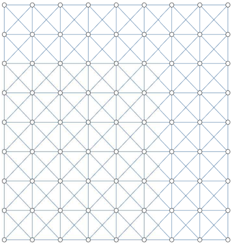
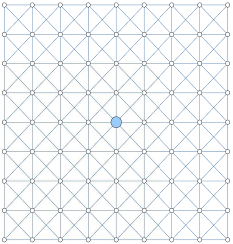
One Good on a Regular Geometry

One Traded Good, Endowment Economy, Output 10x Larger at Center, Uniform Fixed Population



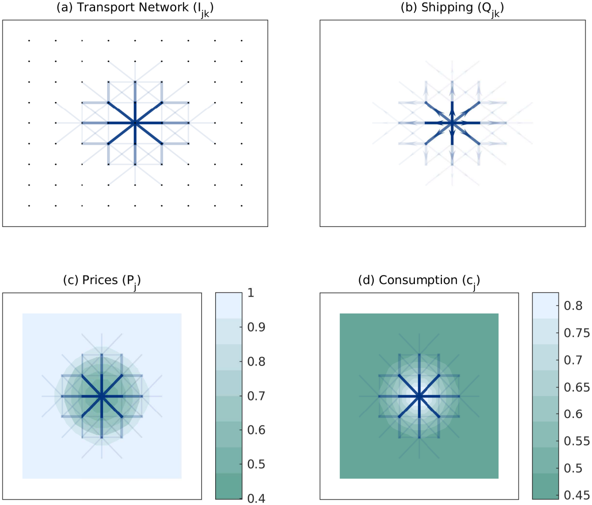
(a) Population



(b) Productivity

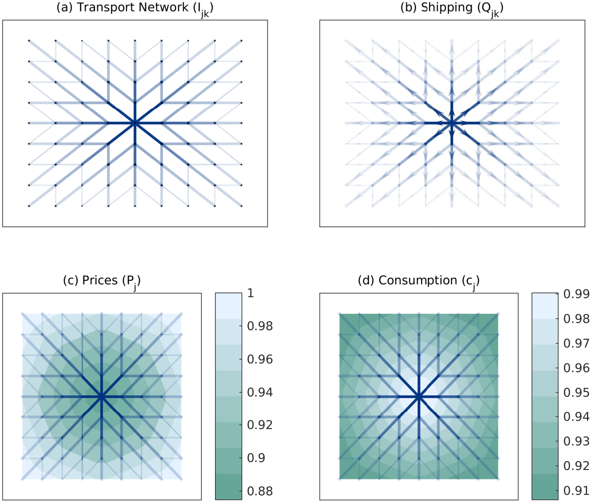
One Good on a Regular Geometry

Optimal Network, $K = 1$



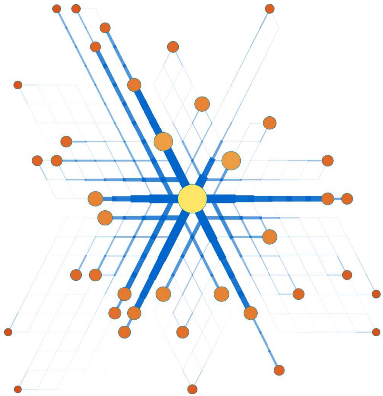
One Good on a Regular Geometry

Optimal Network, $K = 100$



Randomly Located Cities, Convex case

40 randomly cities, central node more productive



Randomly Located Cities, Nonconvex case

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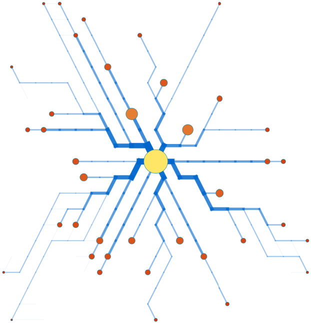
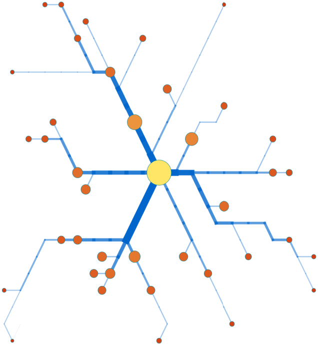


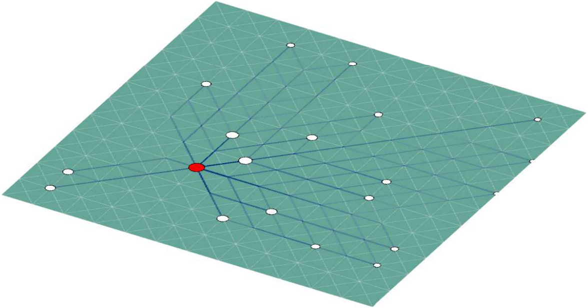
(a) local optimum (FOC)



(b) with annealing

Role of Building Costs

20 random cities across uniform geography, convex case

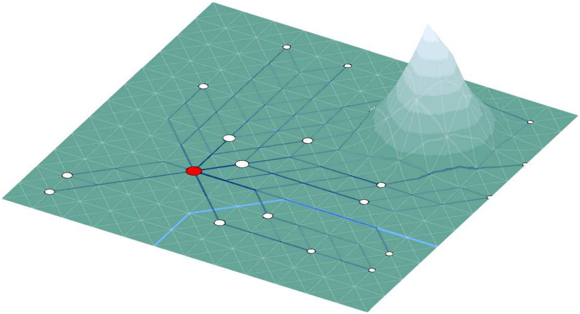


Geography: $\delta^I_{jk} = \delta_0 \text{Distance}^{\delta_1}_{jk}$.

Role of Building Costs

Adding a mountain, a river, bridges, and water transport

Building Cost: $\delta^{I}_{jk} = \delta_0 \text{Distance}_{jk}^{\delta_1} \left(1 + |\Delta\text{Elevation}|_{jk}\right)^{\delta_2} \delta_3^{\text{CrossingRiver}_{jk}} \delta_4^{\text{AlongRiver}_{jk}}$



Plan

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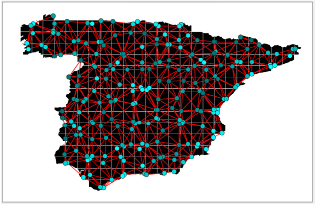
Application

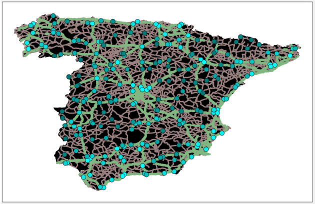
- Questions
  - What are the aggregate gains from optimal expansion of current networks?
  - What would be the regional effects?
  - How important are the inefficiencies in infrastructure investments?
- In 25 European countries we observe
  - Road networks with features of each segment: number of lanes and national/local road (EuroGeographics)
  - Value Added (G-Econ 4.0)
  - Population (GPW)

Underlying Graph and Observed Infrastructure

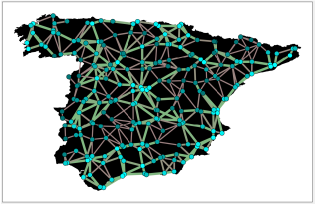
- Construct the graph $(\mathcal{J}, \mathcal{E})$ with quality of actual road network for each country
  - $\mathcal{J}$: population centroids of 0.5 x 0.5 degree ($\sim$50 km) cells
  - $\mathcal{E}$: all links among contiguous cells (8 neighbors per node)
  - I_{jk}^{obs}: observed infrastructure between all connected $jk \in \mathcal{E}$ table all countries



(a) Underlying Graph



(b) Actual Road Network



(c) Measured Infrastructure I_{jk}^{obs}

Calibration

- **Production technologies**: $Y_j^n = z_j^n L_j^n$
- **Preferences**: $U(c, h) = c^{\alpha} h^{1-\alpha}$
  - $N \in \{5, 10, 15\}$ tradeable sectors with CES demand ($\sigma = 5$)
- **Transport technology**: $\tau_{jk}^n = \delta_{jk}^{\tau} \frac{\left(Q_{jk}^n\right)^{\beta}}{I_{jk}^{\gamma}}$
  - Global convexity if $\beta > \gamma$
    - $\beta = 0.13$, $\gamma = 0.10$ from Couture et al. (2018)
    - Also sensitivity with nonconvex case $\gamma = \frac{0.13}{0.10}\beta$
  - Geographic frictions $\delta_{jk}^{\tau} = \delta_0^{\tau}\, dist_{jk}$ matches intra-regional share of intra-national trade in Spain
- **Fundamentals**: $\{z_j, H_j\}$ such that $\left\{GDP_j^{obs}, L_j^{obs}\right\}$ is the model's outcome given $\left\{I_{jk}^{obs}\right\}$
  - Model fit ▸ Fit
  - Trade-distance elasticity (not targeted) close to 1 ▸ Table
- **Building costs** $\delta_{jk}^{I,GEO}$: as function of distance and ruggedness using estimates from Collier et al. (2016)

Optimal Expansion and Reallocation

- Optimal reallocation
  - Fix K equal to calibrated model
  - Build anywhere ($I_{jk} \geqslant 0$)
- Optimal expansion
  - Increase K by 50% relative to calibrated model
  - Build on top of existing network ($I_{jk} \geqslant I_{jk}^{obs}$)

Average Aggregate Effects

Returns to Scale:	Convex		Non-Convex	
Labor:	Fixed	Mobile	Fixed	Mobile
Optimal Reallocation	1.6%	1.5%	2.4%	1.9%
Optimal Expansion	1.7%	1.6%	2.8%	2.2%

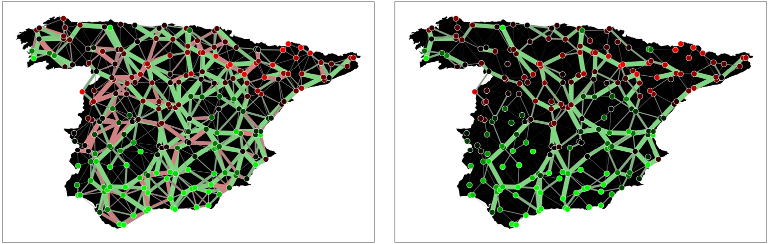
Each element of the table shows the average welfare gain in the corresponding counterfactual across the 24 countries.

Cross-Country Effects

Moblie Labor

ICC	Non-Convex			Convex		
	Expansion (GEO)	Expansion (FOC)	Misallocation	Expansion (GEO)	Expansion (FOC)	Misallocation
Austria	3.0%	1.1%	2.6%	2.1%	0.3%	2.1%
Belgium	0.8%	0.3%	0.7%	0.6%	0.1%	0.5%
Cyprus	1.4%	0.5%	1.2%	0.9%	0.2%	0.9%
Czech Republic	1.5%	0.7%	1.2%	0.9%	0.2%	0.9%
Denmark	7.0%	1.0%	6.6%	3.7%	0.3%	3.7%
Finland	4.0%	2.0%	3.4%	2.5%	0.4%	2.2%
France	3.4%	1.7%	2.6%	3.0%	0.5%	2.8%
Georgia	3.1%	1.6%	2.6%	2.1%	0.3%	2.2%
Germany	2.5%	0.9%	2.1%	1.7%	0.4%	1.5%
Hungary	2.2%	0.7%	1.9%	1.6%	0.2%	1.6%
Ireland	1.6%	1.9%	1.0%	1.4%	0.2%	1.3%
Italy	1.4%	0.6%	0.9%	1.3%	0.6%	1.0%
Latvia	3.6%	1.3%	3.2%	2.3%	0.3%	2.4%
Lithuania	3.0%	1.7%	2.5%	2.3%	0.3%	2.3%
Luxembourg	0.2%	0.2%	0.1%	0.2%	0.1%	0.1%
Macedonia	0.5%	0.3%	0.4%	0.4%	0.1%	0.4%
Moldova	1.8%	0.7%	1.6%	1.0%	0.2%	0.9%
Netherlands	1.5%	0.6%	1.3%	1.1%	0.2%	1.1%
Northern Ireland	0.3%	0.6%	0.4%	0.5%	0.1%	0.5%
Portugal	2.2%	0.9%	1.8%	1.2%	0.3%	1.1%
Slovakia	1.3%	1.4%	0.7%	1.5%	0.3%	1.5%
Slovenia	1.7%	0.8%	1.5%	1.3%	0.1%	1.4%
Spain	4.4%	2.5%	3.9%	3.6%	0.5%	3.4%
Switzerland	1.7%	0.5%	1.5%	1.0%	0.2%	0.9%
Average	2.2%	1.0%	1.9%	1.6%	0.3%	1.5%

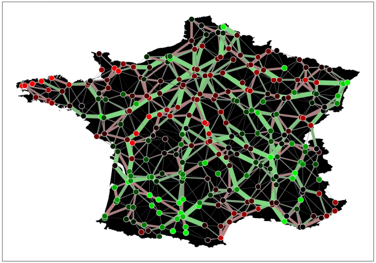
Optimal Reallocation and Expansion: Spain



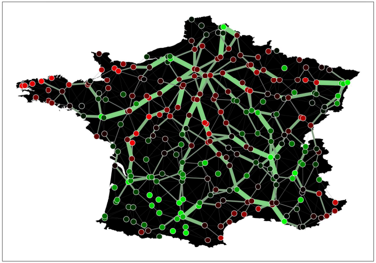
(a) Reallocation

(b) Expansion

Optimal Reallocation and Expansion: France

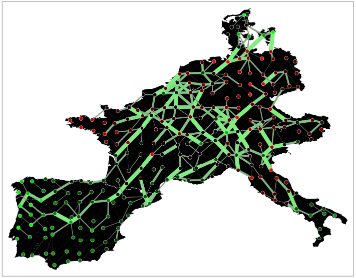


(a) Reallocation

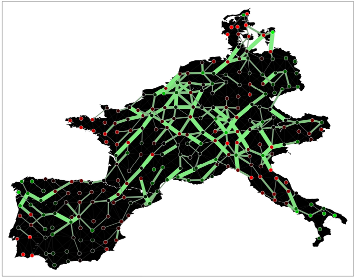


(b) Expansion

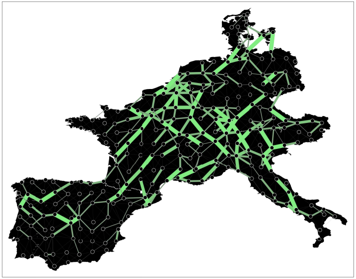
Europe



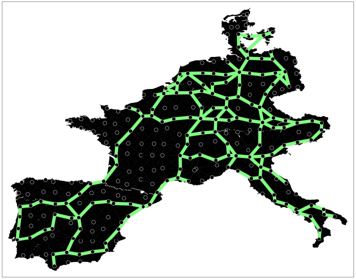
(c) Full Mobility



(d) Labor Mobility within Countries



(e) No Mobility



(f) Discretized TEN-T Network

Where is infrastructure placed?

Dependent variable: Infrastructure growth in the counterfactual

	Reallocation	Expansion
Population	0.358***	0.136***
Income per Capita	0.335***	0.164***
Infrastructure	-0.439***	-0.242***
R^2	0.32	0.27

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.

Which regions grow?

Dependent variable: Population growth in the counterfactual

	Reallocation	Expansion
Population	0.000	-0.001**
Income per Capita	0.026***	0.020***
Consumption per Capita	-0.074***	-0.062***
Infrastructure	-0.001**	-0.001***
Infrastructure Growth	0.001	0.000
Differentiated Producer	0.008**	0.001***
R^2	0.47	0.50

These regressions pool the outcomes across all locations in the convex case.

Country fixed effects included. SE clustered at the country level.

more

Conclusion

- We develop and implement a framework to study optimal transport networks
  1. Neoclassical model (with labor mobility) on a graph
  2. Optimal Transport Flows with congestion
  3. Optimal Network
- Application to road networks in Europe
  - Welfare gains vary between 0.5% to 5%
  - Optimal expansion of current road networks reduces regional inequalities
- Other potential applications
  - Political economy / competing planners
  - Model-based instruments for empirical work on impact of infrastructure
  - Optimal investments in developing countries
  - Optimal transport of workers
  - Absent forces: agglomeration, dynamics

Planner's Problem (Immobile Labor)

Definition

The planner's problem without labor mobility given the infrastructure network is

$$W_0\left(\left\{I_{jk}\right\}\right)=\max_{\mathbf{C}_j,\mathbf{L}_j,\mathbf{V}_j^n,\mathbf{X}_j^n}\max_{Q_{jk}^n}\sum_j \omega_j L_j U\left(c_j,h_j\right)$$

subject to (i) availability of traded and non-traded goods,

$$c_j L_j \le C_j^T\left(\mathbf{C}_j\right) \text{ and } h_j L_j \le H_j \text{ for all } j;$$

(ii) the balanced-flows constraint,

$$C_j^n+\sum_{n'}X_j^{nn'}+\sum_{k\in\mathcal{N}(j)}\left(Q_{jk}^n+\tau_{jk}^n\left(Q_{jk}^n,I_{jk}\right)Q_{jk}^n\right)=F_j^n\left(L_j^n,\mathbf{V}_j^n,\mathbf{X}_j^n\right)+\sum_{i\in\mathcal{N}(j)}Q_{ij}^n \text{ for all } j,n;$$

(iii) local labor-market clearing; and

(iv) factor market clearing and non-negativity constraints.

Return

Example: Log-Linear Transport Technology

- Log-linear transport technology:

$$\tau_{jk}\left(Q, I\right) = \delta_{jk}^{\tau} \frac{Q^{\beta}}{I^{\gamma}}$$

  - Global convexity if $\beta > \gamma$

- Optimal network

$$I_{jk}^{*} \propto \left[\frac{1}{\delta_{jk}^{I} \left(\delta_{jk}^{\tau}\right)^{\frac{1}{\beta}}} \left(\sum_{n: P_k^n > P_j^n} P_j^n \left(\frac{P_k^n}{P_j^n} - 1 \right)^{\frac{1+\beta}{\beta}} \right) \right]^{\frac{\beta}{\beta - \gamma}}$$

  where P_j^n are the equilibrium prices in GE

Return

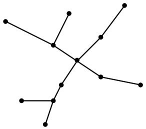
Nonconvex Case: Economies of Scale in Transport

- What if $\gamma > \beta$, i.e., returns to transport technology are increasing?
  - KKT no longer sufficient: local vs. global optimality
- Computations show the network becomes extremely sparse
  - The planner prefers to concentrate flows on few large "highways" (branched transport)

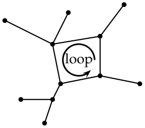
Proposition

(**Network Shape in Non-Convex Cases**) *In the absence of a pre-existing network (i.e., $I^0_{jk} = 0$), if the transport technology is satisfies $\gamma > \beta$ and there is a unique commodity produced in a single location, the optimal transport network is a tree.*

- Intuition: cycles are sub-optimal because it pays off to remove an edge and concentrate flows elsewhere



(a) tree (no loops)

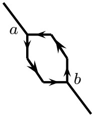


(b) non-tree

Return

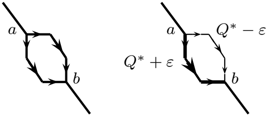
Tree Property with Economies of Scale: Intuition

- Two types of elementary cycles can occur:



Case (i) Case (ii)

- **Argument:**
  - Cycles of type (i) waste resources and can be ruled out
  - With cycles of type (ii), better off to redirect flows to one branch:



(a) Initial (b) After redirection

Fictitious Planner Problem with Externalities

- We may assume that the market allocation is inefficient, e.g. due to
  - No corrective taxes: $t_{jk}^n = 0$
  - Externalities from population: $F_j^n\left(\cdot; L_j\right)$

Definition

The fictitious-planner's problem with externalities given the infrastructure network is

$$W_0\left(\{I_{jk}\}; \overline{L}, \overline{Q}\right) = \max_{\mathbf{C}_j, \mathbf{L}_j, \mathbf{V}_j^n, \mathbf{X}_j^n, L_j} \max_{Q_{jk}^n} u$$

subject to conditions (i), (iii), (iv), (v) from before, and

$$C_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(Q_{jk}^n + \tau_{jk}^n\left(\overline{Q_{jk}^n}, I_{jk}\right) Q_{jk}^n\right) = F_j^n\left(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n; \overline{L}_j\right) + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \text{ for all } j,\ n.$$

- This problem satisfies the same convexity+duality properties of the standard planner's given the network.
- Letting $\mathbf{L}_0\left(\overline{L}, \overline{Q}\right)$ and $\mathbf{Q}_0\left(\overline{L}, \overline{Q}\right)$ be the solution to the fictitious-planner problem with externalities:

Proposition

(Decentralization with Externalities) $\left(\overline{L}, \overline{Q}\right)$ *corresponds to an inefficient market allocation if and only if* $\overline{L} = \mathbf{L}_0\left(\overline{L}, \overline{Q}\right)$ *and* $\overline{Q} = \mathbf{Q}_0\left(\overline{L}, \overline{Q}\right)$.

Return

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Return

Computation: Convex Case

- The convex case $\beta \geq \gamma$ is well-behaved but still high-dimensional. At worst:
  - $N \times J^2$ shipped quantities Q^n_{jk},
  - J^2 variables I_{jk} (fully connected graph)
  - $(N+1) \times J$ aggregate consumption levels C^n_j, C_j
  - $N \times J + 1$ Lagrange multipliers P^n_j, μ
  - $N \times L$ employment levels L^n_j with labor mobility

- Convexity ensures that efficient gradient-descent methods converge
  - Powerful free/open-source convex solvers based on interior-point methods such as IPOPT, CVXOPT, GAMS... can handle very large systems
  - Exploit sparsity of underlying geography to speed up calculations
  - Exploit the Lagrangian duality approach preferred in optimal transport problems (Bertsekas, 1998)

Return

Computation: Primal vs Dual

- Primal approach

$$\sup_{C,L,Q} \inf_{P} \mathscr{L}\left(C, L, Q; P\right)$$

  - Even if convex, slow to converge (high dimension in C, L, Q)

- Dual approach

$$\inf_{P} \mathscr{L}\left(C\left(P\right), L\left(P\right), Q\left(P\right); P\right)$$

  - Use FOCs and substitute for C, Q,..., as function of P, then minimize over Lagrange multipliers
  - Convex minimization problem in fewer variables with just non-negativity constraints (just P)

Return

Computation: Nonconvex Case

- Our approach
  - Given the network, solve the he optimal transport+neoclassical allocation using primal/duality
  - Iterate on the network FOCs:

$$I_{jk} = \left(\frac{\gamma}{\mu} \frac{\delta^{\tau}_{jk}}{\delta^{I}_{jk}} \sum_{n} P^{n}_{j} \left(Q^{n}_{jk} \right)^{1+\beta} \right)^{\frac{1}{1+\gamma}}$$

  - Always converges in practice, but towards a local optimum ($\sim$lock in?)
  - We thus combine this approach with home-made algorithm based on simulated annealing
    - Simulated annealing is an easy-to-implement heuristical approach
    - Often used for large combinatorial problems such as traveling salesman, etc.
    - Improves results, but no guarantee of convergence to global maximum

Return

Discretization of the Observed Network

- Road network from EuroGeographics data provides the following characteristics:
  - Number of lanes, national/primary/secondary road, paved/unpaved, median, etc.
- For each pair of locations (j,k), we identify the *least cost route* on the observed network using relative user costs from Combes and Lafourcade (2005)
- In the discretized network, we construct an infrastructure index

$$I_{jk} = lanes_{jk} \times \chi_{nat}^{1-nat_{jk}}$$

where

  - $lanes_{jk}$ is the average observed number of lanes on the least cost route from j to k
  - $nat_{jk} \in [0, 1]$ is the fraction of that route spent on national roads
  - χ_{nat} is the building and maintenance cost per km. of national roads relative to other types (Doll et al., 2008)

Return

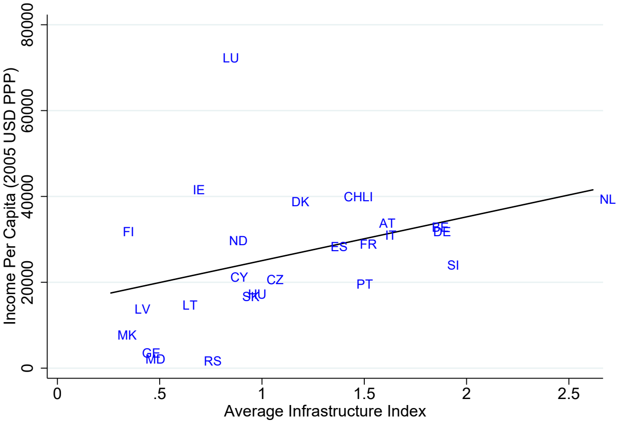
Average Infrastructure Across Countries

Country	Code	Actual Road Network			Discretization		
		Length (Km.)	Number of Segments	Average Lanes per Km.	Number of Cells	Length (Km.)	Average Infrastructure Index
		(1)	(2)	(3)	(4)	(5)	(6)
Austria	AT	17230	6161	2.36	46	9968	1.54
Belgium	BE	19702	10496	2.49	20	3400	1.80
Cyprus	CY	2818	947	2.29	30	3653	0.81
Czech Republic	CZ	28665	10194	2.20	48	10935	0.99
Denmark	DK	11443	4296	2.18	21	4102	1.11
Finland	FI	70394	9221	2.04	73	34262	0.29
France	FR	128822	38699	2.05	276	78405	1.45
Georgia	GE	28682	9009	1.95	32	7895	0.38
Germany	DE	115177	66428	2.42	196	56410	1.80
Hungary	HU	32740	9244	2.10	50	12017	0.90
Ireland	IE	24952	4144	2.10	47	11299	0.63
Italy	IT	77608	44159	2.32	126	32640	1.57
Latvia	LV	11495	2103	2.03	47	10599	0.34
Lithuania	LT	10682	1586	2.39	43	9736	0.58
Luxembourg	LU	1779	866	2.31	8	674	0.78
Macedonia	MK	5578	908	2.15	14	2282	0.26
Moldova	MD	8540	1462	2.21	20	4611	0.40
Netherlands	NL	14333	8387	2.68	20	4263	2.62
Northern Ireland	ND	7087	1888	2.18	12	1714	0.81
Portugal	PT	15034	4933	2.10	43	11000	1.43
Serbia	RS	18992	3656	2.14	45	12261	0.68
Slovakia	SK	11420	2610	2.18	30	6130	0.87
Slovenia	SI	7801	2441	2.19	12	1853	1.87
Spain	ES	101990	18048	2.39	227	68162	1.30
Switzerland	CHLI	14526	11102	2.26	25	5127	1.37

- Infrastructure index:
  - Across countries: correlated with income per capita fig
  - Within countries: 0.7 correlation with travel times (GoogleMaps) across all pairs fig

return

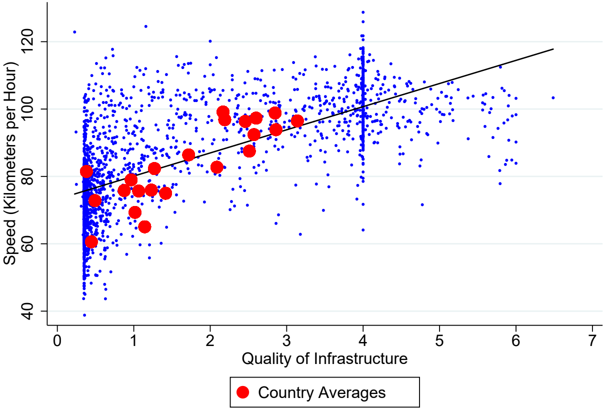
Average Infrastructure and Income Per Capita



Linear regression slope (robust SE): 10200.4 (3603.5)

back

Average Infrastructure and Speed



Regression slope (robust SE): 5.445 (.188). Pools all links. Includes country fixed effects.

back

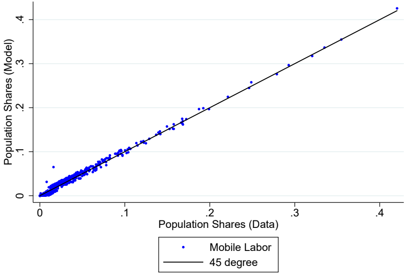
Trade-Distance Elasticity

Value of γ:	0.5β		β		1.5β	
Labor:	Fixed	Mobile	Fixed	Mobile	Fixed	Mobile
Average	-1.08	-1.11	-1.12	-1.19	-1.16	-1.17
Standard deviation	0.17	0.21	0.17	0.26	0.22	0.27

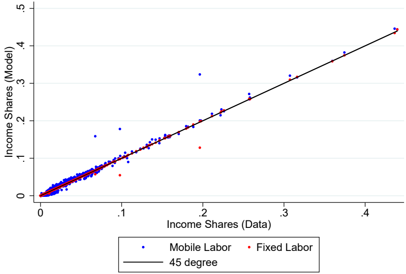
The table reports the average and standard deviation of the trade-distance and elasticity and intra-regional trade share across the 25 countries in our data.

back

Model Fit



Linear regression slope (robust SE): Mobile Labor: 1.026 (.005)

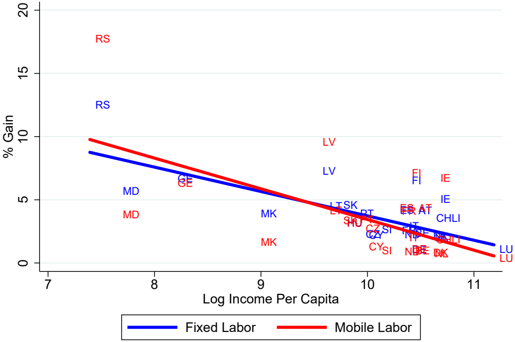


Linear regression slope (robust SE): Mobile Labor: 1.04(.012); Fixed Labor: .995 (.006)

back

Cross-Country Effects

Optimal Expansion, δ^{GEO}

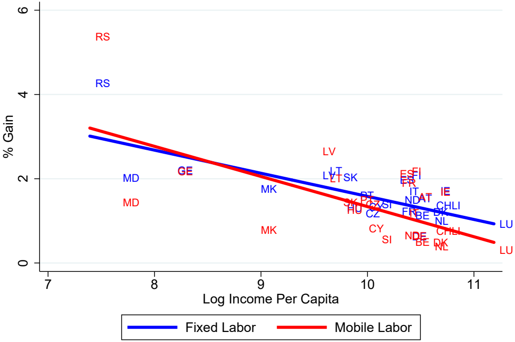


Linear regression slope (robust SE): Mobile Labor: -2.427 (1.209); Fixed Labor: -1.928 (.584)

return

Cross-Country Effects

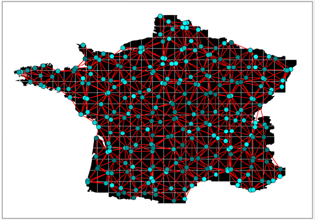
Optimal Expansion, δ^{FOC}



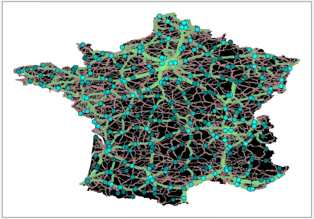
Linear regression slope (robust SE): Mobile Labor: -.715 (.337); Fixed Labor: -.549 (.191)

return

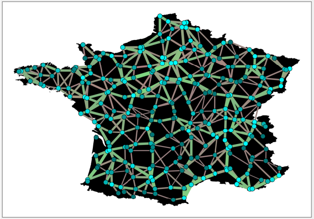
Example: France



(a) Underlying Graph $(\mathcal{J}, \mathcal{E})$



(b) Actual Road Network



(c) Measured Infrastructure I_{jk}^{obs}

Back