The Journal of Open Source Software

StateSpaceDynamics.jl: A Julia package for probabilistic state space models (SSMs)

Ryan Senne [1,2], **Zachary Loschinskey** [1], **James Fourie** [1], **Carson Loughridge** [1], **and Brian D. DePasquale** [1,2]

1 Department of Biomedical Engineering, Boston University **2** Graduate Program for Neuroscience, Boston University

DOI: 10.xxxxxx/draft

Software

- Review
- Repository
- Archive

Editor: Open Journals

Reviewers:

- @openjournals

Submitted: 01 January 1970
Published: unpublished

Summary

State-space models (SSMs) are powerful tools for modeling time series data that naturally arise in a variety of domains, including neuroscience, finance, and engineering. The unifying principle of these models is they assume an observation sequence, $Y_1, Y_2, ..., Y_T$, is generated through an underlying Markovian latent sequence, $X_1, X_2, ..., X_T$. This framework encompasses two popular models for time series analysis: the hidden Markov model (HMM) and the (Gaussian) linear dynamical system (LDS, i.e., the Kalman filter). Thus, SSMs provide a probabilistic framework for describing the temporal evolution of many phenomena, and their generality naturally leads to a variety of use cases. We introduce `StateSpaceDynamics.jl` (Senne et al., 2025), an open-source, modular package designed to be fast, readable, and self-contained for the purpose of easily fitting a plurality of SSMs in the Julia language.

Statement of need

Advances in neuroscience have enabled the collection of massive, multivariate, and complex time-series datasets, where simultaneous observations from hundreds to thousands of neurons are increasingly common. Interpreting these high-dimensional datasets presents significant challenges. Recent modeling approaches suggest that neural activity can be characterized by a set of latent factors evolving within a low-dimensional manifold. Consequently, there is a growing need for models that combine dimensionality reduction with temporal dynamics, for which state-space models provide a natural framework.

While state-space model implementations exist in Python, such as the `ssm` package (S. Linderman, 2022) and `Dynamax` (Scott W. Linderman et al., 2025), the Julia programming language lacks an equivalent that meets the needs of modern neuroscience. Existing Julia offerings, like `StateSpaceModels.jl` (Saavedra et al., 2019), can accommodate continuous-state SSMs (e.g., LDS) but are limited to Gaussian observation models and rely on analytical calculation of the marginal log-likelihood. This latter limitation precludes model inference and parameter learning for non-conjugate observations which are common in neuroscience, where neural activity follow Poisson or other discrete distributions. Packages for performing inference and learning using sampling-based methods exist in Julia (such as `Turing.jl` (Fjelde et al., 2025; Ge et al., 2018)) but are computationally inefficient compared to tailored approaches based on Expectation-Maximization (EM). For discrete SSMs, an existing Julia offering, `HiddenMarkovModels.jl` (Dalle, 2024), is efficient and scalable but not intentionally designed with the functionality for mixing models that contain both discrete and continuous latent variables, such as the switching linear dynamical system model (SLDS) (Ghahramani & Hinton, 2000; Scott W. Linderman et al., 2016) increasingly used in neuroscience. Although our primary motivation arises from challenges in modeling high-dimensional neural population activity, the package is not specific to neuroscience. The algorithms and abstractions apply equally well to time-series problems in engineering, econometrics, and other fields where latent variable models and structured inference are required.

Package design

To address these limitations, we developed `StateSpaceDynamics.jl`, which provides a flexible framework for fitting a variety of SSMs–including non-Gaussian observation models and models that mix discrete and continuous latents–while maintaining computational efficiency.

For continuous latent-variable models, (e.g., LDS) `StateSpaceDynamics.jl` employs a previously advocated approach of directly maximizing the complete-data log-likelihood with respect to the hidden state path (Paninski et al., 2010). By leveraging the block tridiagonal structure of the Hessian matrix, this method allows for the exact computation of the Kalman smoother in $\mathcal{O}(T)$ time (Paninski et al., 2010). Furthermore, it facilitates the generalization of the Rauch–Tung–Striebel (RTS) smoother to accommodate other observation models (e.g., Poisson and Bernoulli), requiring only the computation of the gradient and Hessian of the new model to obtain an *exact* maximum a posteriori (MAP) path (Macke et al., 2011).

Using analytically computable Hessians, `StateSpaceDynamics.jl` performs approximate EM for non-Gaussian models via Laplace approximation of the latent posterior. Speed is maintained by using fast inversion algorithms of the negative Hessian (i.e., Fisher Information Matrix), which are block tridiagonal (Rybicki & Hummer, 1990). From here `StateSpaceDynamics.jl` computes the approximate second moments of the posterior i.e., Cov(X_t, X_t) and Cov(X_t, X_{t-1}), and uses the analytical updates of the canonical LDS (Bishop, 2006; Paninski et al., 2010). It is important to note that when the observations and state-evolution process are assumed to have Gaussian errors, this approach is exactly the same as using the standard Kalman Filter and RTS-Smoother, i.e., they will give the same results.

Lastly, `StateSpaceDynamics.jl` provides implementations of discrete state-space models i.e., hidden Markov models, and the ability to fit these models using EM. While this is not the primary development target of the package, these models are necessary for the development of hierarchical models that mix discrete and continuous latents, e.g., the switching LDS (SLDS) and the recurrent switching LDS (rSLDS) (Ghahramani & Hinton, 2000; Scott W. Linderman et al., 2016; Murphy, 1998) which have become immensely popular in neuroscience and require similarly tailored computational routines for efficient inference and learning. To illustrate the functionality of `StateSpaceDynamics.jl` for this model class, we include an implementation of Variational Laplace EM (vLEM) (Zoltowski et al., 2020). The development of `HiddenMarkovModels.jl`, may make our approach to discrete model learning redundant, and future work may entail directly interfacing with this package (Dalle, 2024). Nonetheless, we provide a suite of HMM models popular in neuroscience including the classic Gaussian HMM and a variety of input-output HMMs (Bengio & Frasconi, 1994), commonly referred to as generalized linear model-HMMs (GLM-HMMs) (Ashwood et al., 2022) in neuroscience.

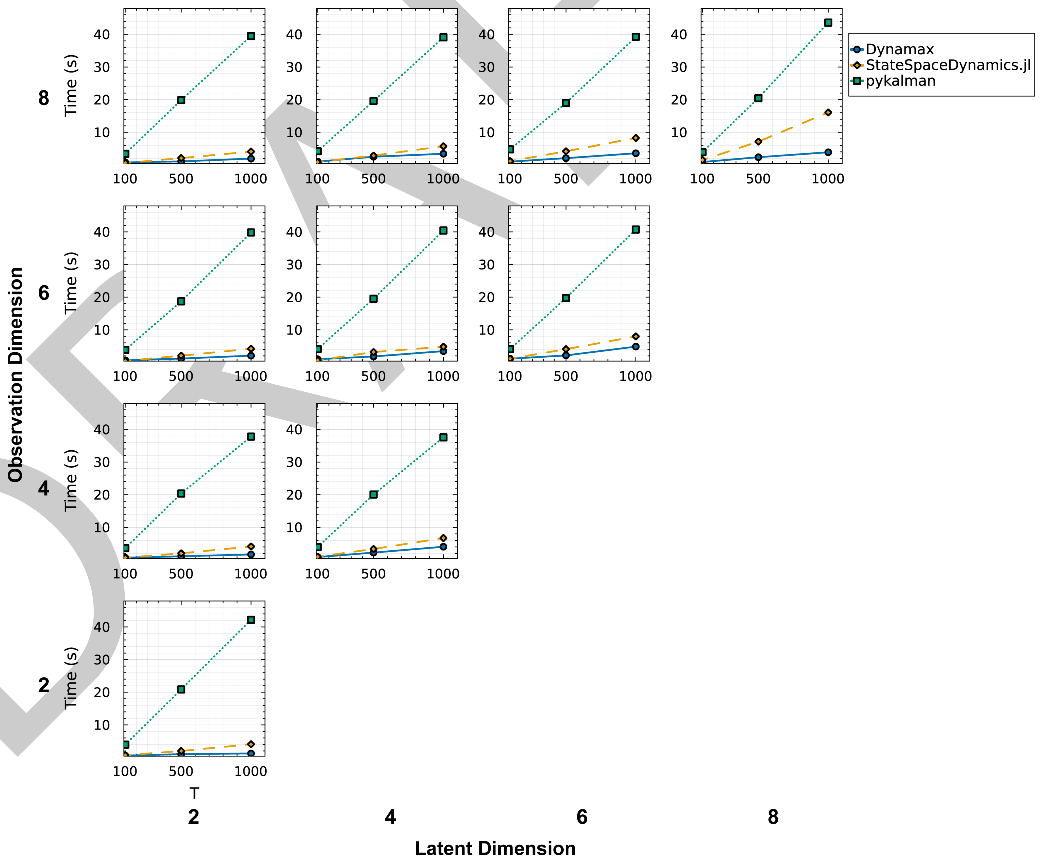
By providing these features, `StateSpaceDynamics.jl` fills a critical gap in the Julia ecosystem, offering modern computational neuroscientists the tools to model complex neural data with state-space models that incorporate both dimensionality reduction and temporal dynamics.

Benchmarks

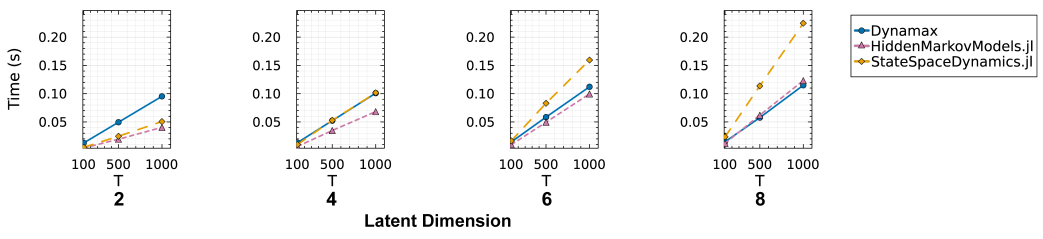
To evaluate the performance of `StateSpaceDynamics.jl`, we conducted two benchmarking studies focusing on fitting a Gaussian LDS and a Gaussian HMM. For the Gaussian LDS benchmark, we compared our package against two alternatives: the NumPy-based Kalman filter-smoother package `pykalman` and the more recent JAX-based `Dynamax`. We intentionally excluded `StateSpaceModels.jl` from our comparison as its scope is geared towards structured time-series models. Dynamax was properly JIT-compiled using the jax.jit function prior to benchmarking to ensure fair comparison.

For our Gaussian LDS experiments, we constructed a synthetic dataset as follows. The state transition matrix A was generated as a random n-dimensional rotation matrix, while the observation matrix C was created as a random $m \times n$ matrix. Both the state noise covariance Q and observation noise covariance R were set to identity matrices. To ensure a fair comparison, all packages were initialized using identical random parameters, after which we executed the EM algorithm for 100 iterations. We conducted these benchmarks using PythonCall.jl (Doris, 2021) and BenchmarkTools.jl (Chen & Revels, 2016), with the assumption that Julia-to-Python overhead is negligible for these computationally intensive operations.

To thoroughly assess performance across different scales, we tested three sequence lengths ($T = 100, 500, 1000$) and explored multiple dimensionality settings, with state dimensions $n = 2, 4, 6, 8$ and observation dimensions $m = 2, 4, 6, 8$. In all cases, we restricted evaluations to settings where the latent dimension was less than or equal to the observation dimension. Finally, it is worth noting that Dynamax includes a temporally parallel smoother with $\mathcal{O}(\log T)$ complexity. We did not include this method in our comparisons because it is GPU-specific and incompatible with our direct optimization approach, which is designed for inference in non-conjugate models.



For the second benchmarking study, we compared StateSpaceDynamics.jl, HiddenMarkovModels.jl, and Dynamax in their ability to fit a Gaussian HMM. Once again, we ensured that Dynamax was JIT-compiled for a fair comparison. To construct synthetic datasets, we sampled from a Gaussian HMM with randomly selected emission models, transition matrices, and initial state distributions. Each package was initialized using identical random parameters to maintain consistency. EM was run for 100 iterations.



In our benchmarking, we find that for the LDS, both `StateSpaceDynamics.jl` and `Dynamax` are faster than `pykalman` across all sequence lengths and dimension configurations. More generally, `StateSpaceDynamics.jl` and `Dynamax` exhibit similar performance at lower sequence lengths (with `Dynamax` slightly outperforming `StateSpaceDynamics.jl`). However, `Dynamax` exhibits superior scaling in both the dimensions of the state and observation matrices as well as the temporal sequnce length. In our current implementation, the Hessian is represented as a sparse matrix with block tridiagonal structure, resulting in $\mathcal{O}(Tn^2)$ memory scaling — which is optimal. However, we do not yet exploit this structure fully during inference. In particular, our solver does not leverage specialized routines for block-banded systems (e.g., the block Thomas algorithm), which can result in unnecessary fill-in and degraded performance at large T. Future versions will use banded or block tridiagonal solvers to achieve truly linear-time inference.

In our HMM benchmarks, `HiddenMarkovModels.jl` outperforms both `StateSpaceDynamics.jl` and `Dynamax` across most sequence lengths and state dimensions, with `Dynamax` only becoming slightly faster for high state dimensions and long sequence lengths. `StateSpaceDynamics.jl` outperforms `Dynamax` at low state dimensions for all sequence lengths but exhibits worse scaling with the number of states, allowing `Dynamax` to overtake it as the number of states increases. These results, combined with our primary development goals in hierarchical SSMs, highlight the benefits of interfacing with `HiddenMarkovModels.jl` for HMM-specific functionality. Efforts are currently underway to make this interface seamless.

Taken together, these benchmarks demonstrate the competitiveness of `StateSpaceDynamics.jl` for fitting state-space models. Our benchmarks are available in the `benchmarking` folder of our repository, and instructions for running these are available in a `README.md` file.

Availability

`StateSpaceDynamics.jl` is publicly available under the GNU license at https://github.com/depasquale-lab/StateSpaceDynamics.jl.

Future Directions

The current release of `StateSpaceDynamics.jl` emphasizes efficient CPU-based implementations and analytically derived gradients and Hessians for commonly used observation models. Several avenues of future development will broaden the scope and accessibility of the package. First, we plan to add optional support for automatic differentiation (AD) using Julia's AD ecosystem (e.g., `ForwardDiff.jl` (Revels et al., 2016), `Zygote.jl` (Innes, 2018), `DifferentiationInterface.jl` (Dalle & Hill, 2025)). This will allow users to prototype new observation models without requiring hand-coded derivatives, while maintaining the existing optimized implementations for speed-critical cases. Second, we aim to extend hardware support to GPU backends by exploiting Julia's GPU array abstractions and block-tridiagonal solvers, enabling large-scale inference with temporally parallel methods. Finally, we plan to expand parameter inference options beyond maximum likelihood and Laplace-EM, including Bayesian approaches via variational inference and interoperability with probabilistic programming frameworks such as `Turing.jl`. Together, these developments will further enhance the package's flexibility, performance, and utility across scientific disciplines.

Conclusion

`StateSpaceDynamics.jl` fills an existing gap in the Julia ecosystem for general state-space modeling that exists in Python. Importantly, our package's approach is simple enough that other candidate state-space models can be easily implemented. Further, this work provides a foundation for future development of more advanced state-space models, such as the rSLDS, which are essential for modeling complex neural data. We expect that this package will be of interest to computational neuroscientists and other researchers working with high-dimensional time series data and we are currently using its functionality in three separate projects.

Author contributions

RS (Ryan Senne) was the primary developer of `StateSpaceDynamics.jl`, implementing the core algorithms, designing the package architecture, and writing the manuscript. ZL (Zachary Loschinskey) was the secondary developer, whose contributions include optimizing and extending HMM/GLM-HMM functionality, implementing core multi-trial EM algorithms, and assisting with SLDS development. CL (Carson Loughridge) and JF (James Fourie) contributed to package development, including implementation of key features, testing, and documentation. BDD (Brian D. DePasquale) conceived the project, provided theoretical guidance and technical oversight throughout development, secured funding, and supervised the work. All authors reviewed and approved the final manuscript.

Acknowledgements

This work was supported by the Biomedical Engineering Department at Boston University.

References

Ashwood, Z. C., Roy, N. A., Stone, I. R., Urai, A. E., Churchland, A. K., Pouget, A., & Pillow, J. W. (2022). Mice alternate between discrete strategies during perceptual decision-making. *Nature Neuroscience*, *25*(2), 201–212. https://doi.org/10.1038/s41593-021-01007-z

Bengio, Y., & Frasconi, P. (1994). An input output HMM architecture. In G. Tesauro, D. Touretzky, & T. Leen (Eds.), *Advances in neural information processing systems* (Vol. 7). MIT Press. https://proceedings.neurips.cc/paper_files/paper/1994/file/8065d07da4a77621450aa84fee5656d9-Paper.pdf

Bishop, C. M. (2006). *Pattern recognition and machine learning*. Springer.

Chen, J., & Revels, J. (2016). Robust benchmarking in noisy environments. *arXiv e-Prints*. https://arxiv.org/abs/1608.04295

Dalle, G. (2024). HiddenMarkovModels.jl: Generic, fast and reliable state space modeling. *Journal of Open Source Software*, *9*(96), 6436. https://doi.org/10.21105/joss.06436

Dalle, G., & Hill, A. (2025). *A common interface for automatic differentiation*. https://arxiv.org/abs/2505.05542

Doris, C. (2021). *PythonCall.jl*. https://github.com/JuliaPy/PythonCall.jl.

Fjelde, T. E., Xu, K., Widmann, D., Tarek, M., Pfiffer, C., Trapp, M., Axen, S. D., Sun, X., Hauru, M., Yong, P., Tebbutt, W., Ghahramani, Z., & Ge, H. (2025). Turing.jl: A general-purpose probabilistic programming language. *ACM Trans. Probab. Mach. Learn.* https://doi.org/10.1145/3711897

Ge, H., Xu, K., & Ghahramani, Z. (2018). Turing: A language for flexible probabilistic inference. In A. Storkey & F. Perez-Cruz (Eds.), *Proceedings of the twenty-first international* *conference on artificial intelligence and statistics* (Vol. 84, pp. 1682–1690). PMLR. https://proceedings.mlr.press/v84/ge18b.html

Ghahramani, Z., & Hinton, G. E. (2000). Variational learning for switching state-space models. *Neural Computation*, *12*(4), 831–864. https://doi.org/10.1162/089976600300015619

Innes, M. (2018). Don't unroll adjoint: Differentiating SSA-form programs. *CoRR*, *abs/1810.07951*. http://arxiv.org/abs/1810.07951

Linderman, S. (2022). *SSM: Bayesian learning and inference for state space models*. https://github.com/lindermanlab/ssm

Linderman, Scott W., Chang, P., Harper-Donnelly, G., Kara, A., Li, X., Duran-Martin, G., & Murphy, K. (2025). Dynamax: A Python package for probabilistic state space modeling with JAX. *Journal of Open Source Software*, *10*(108), 7069. https://doi.org/10.21105/joss.07069

Linderman, Scott W., Miller, A. C., Adams, R. P., Blei, D. M., Paninski, L., & Johnson, M. J. (2016). Recurrent switching linear dynamical systems. *arXiv [Stat.ML]*. https://doi.org/10.48550/arXiv.1610.08466

Macke, J. H., Buesing, L., Cunningham, J. P., Yu, B. M., Shenoy, K. V., & Sahani, M. (2011). *Empirical models of spiking in neural populations*. *24*. https://proceedings.neurips.cc/paper_files/paper/2011/file/7143d7fbadfa4693b9eec507d9d37443-Paper.pdf

Murphy, K. P. (1998). *Switching kalman filters* [Technical report].

Paninski, L., Ahmadian, Y., Ferreira, D. G., Koyama, S., Rahnama Rad, K., Vidne, M., Vogelstein, J., & Wu, W. (2010). A new look at state-space models for neural data. *J. Comput. Neurosci.*, *29*, 107–126. https://doi.org/10.1007/s10827-009-0179-x

Revels, J., Lubin, M., & Papamarkou, T. (2016). Forward-mode automatic differentiation in Julia. *arXiv:1607.07892 [Cs.MS]*. https://arxiv.org/abs/1607.07892

Rybicki, G. B., & Hummer, D. G. (1990). *Fast solution for the diagonal elements of the inverse of a tridiagonal matrix*.

Saavedra, R., Bodin, G., & Souto, M. (2019). StateSpaceModels.jl: A julia package for time-series analysis in a state-space framework. *arXiv Preprint arXiv:1908.01757*.

Senne, R., Loschinskey, Z., Loughridge, C., & DePasquale, B. (2025). *StateSpaceDynamics.jl: A julia package for probabilistic state space models*. https://github.com/depasquale-lab/StateSpaceDynamics.jl. https://github.com/depasquale-lab/StateSpaceDynamics.jl

Zoltowski, D., Pillow, J., & Linderman, S. (2020). A general recurrent state space framework for modeling neural dynamics during decision-making. In H. D. III & A. Singh (Eds.), *Proceedings of the 37th international conference on machine learning* (Vol. 119, pp. 11680–11691). PMLR. https://proceedings.mlr.press/v119/zoltowski20a.html