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Applications of contour dynamics to two layer quasigeostrophic flows

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Abstract. We determine stationary states and examine dynamic mergers of isolated piecewise constant regions of potential vorticity in a two-layer quasigeostrophic model. We focus on the behavior of the critical initial separation distance for merger, d_c, as a function of γ^{-1} (inverse Rossby radius) and δ, the ration of layer depths.

The two layer quasigeostrophic model has provided valuable insights for geophysical flows, for it is the simplest model that includes both stratification and rotation [1]. The streamfunctions Ψ_1 and Ψ_2 of the upper and lower layer of depths D_1 and D_2, respectively, satisfy the potential vorticity conservation equations

$$\mathrm{d}_{it}\Pi_i = 0 \quad (i = 1, 2), \tag{1}$$

where

$$\mathrm{d}_{it} \equiv \{\partial_t + J(\Psi_i, \cdot)\} \quad \text{and} \quad J(a, b) = a_x b_y - a_y b_x, \tag{2}$$

$$\Pi_1 = \Delta\Psi_1 + \gamma^2(\Psi_2 - \Psi_1), \tag{3}$$

$$\Pi_2 = \Delta\Psi_2 + \delta\gamma^2(\Psi_1 - \Psi_2), \tag{4}$$

and $\delta = D_1/D_2$ is the ratio of the layer depths, and γ is inversely proportional to the Rossby radius and measures the strength of the rotation.

We generalize the contour dynamical method and examine steady-state configurations and evolutions of piecewise-constant (pc) Π_i as discussed in references [2,3]. The velocity of any point in the field is obtained by integrating the total Green's function around contours bounding the regions of pc Π_i. The total Green's function in a layer is obtained from equations (3) and (4) and is the sum of two components which relate the potential velocity in layer j to the vorticity in layer i. For finite δ, the limits $\gamma \to 0$ and $\gamma \to \infty$ yield Euler-like (logarithmic) Green's function. However, for $\delta = 0$ and $\Pi_2 = \Psi_2 = 0$, we recover the *equivalent barotropic model* with a $K_0(\gamma r)$ Green's function.

For $\Pi_2 = 0$, we have found two-fold symmetric ($m = 2$) rotating and also translating stationary states. The rotating states at $\delta = 0$ are "peanut" shaped for large γ and large aspect ratios. However, for small-but-finite δ ($\delta > 0.1$), the states are close to Kirchhoff ellipses over a large-γ range ($10^{-2} < \gamma < 10^2$). Thus, the equivalent barotropic case is a singular limit of the two-layer model and should be applied with caution.

This difference between zero and finite δ is also observed in the *merger* of two identical initially circular regions (radius = 1.0) of potential vorticity in the upper layer and $\Pi_2 = 0$. The initial *critical separation distance* for $\delta = 0$ (equivalent barotropic) begins at 2.0 for $\gamma^{-1} \to 0$ (i.e., the vortices must be relatively close to merge) and approaches the barotropic value

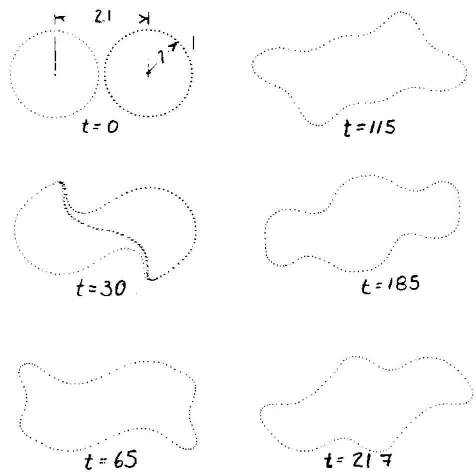
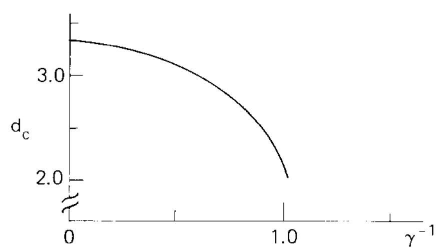


Fig. 1. Merger of two like-signed regions of potential vorticity in the equivalent barotropic model ($\delta = 0$, $\Pi_2 = 0$, $\gamma = 5$) and $d_c = 2.1$. The common contour was "removed" and the remaining single contour smoothed at $t = 25$, prior to continuing. Note the strong nonlinear waves on the surface and no apparent breaking at this resolution.

$d_c \to 3.3$ as $\gamma^{-1} \to \infty$, with a transition at $\gamma^{-1} = 1$. For $\delta = 1$, d_c vs. γ^{-1} is essentially constant at $d_c \approx 3.1$. The latter result has yet to be reconciled with the Griffiths and Hopfinger [4] experiment.

The filamentation (enstrophy cascade) following a merger of two potential-vorticity regions also exhibits this difference. Namely, for $\delta = 0$ and large-γ, axisymmetrization is suppressed following a merger, as shown in fig. 1. That is, for the discretization interval used, the smooth merged contour supports large-amplitude nonfilamenting waves.

Finally, for $\delta = 1.0$ the merger of two like-signed vortices, one in *each* layer, is examined and two qualitatively distinct regions are observed. First, for large-γ the merger is similar to barotropic (Euler equation) merger, i.e., the vortices act as if they were in the same layer. However, for small-γ the interface is "rigid" and the two vortices act nearly independently. This leads to an extreme point in the plot of d_c vs. γ^{-1} at $\gamma \approx 1.0$, as shown in the sketch. Note at $d_c = 2.0$ the curves touch and for $d_c < 2$ they overlap slightly.



We will discuss physically relevant observational consequences of these mergers.

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Noted added in proof

D.G. Dristschel (private communication) had indicated that when very-high resolution contours approach closely in a one-layer quasigeostrophic merger, fine filaments tend to form near high-curvature regions. Hence, our results are "regularized", to a length-scale consistent with our discretization interval. This permits us to make computationally efficient long-time calculations after the common boundary has been removed and smooth contours remain. Furthermore, in recent work, we have interpreted the inhibition of axisymmetrization by examining the critical points of the flow in a proper corotating frame of reference. We find that these points remain outside the vorticity contours for very long times.

We have continued our studies of two-layer mergers and generalized the criteria for merger for $d_c < 2$. As previously, for a given γ^{-1}, we do a sequence of numerical experiments and obtain a small *interval* in d_c across which merger occurs. We find that a d_c vs. γ^{-1} "curve" interpolated through this interval continues below $d_c = 2.0$ and moves toward $\gamma^{-1} = 0$. Thus, we obtain a two-valued "curve" which does not extend beyond $\gamma \approx 1.0$. Furthermore, its shape is in qualitative agreement with the two-valued curve for the *existence* of corresponding two-layer corotating V-states. This qualitative correspondence between merger and existence of V-states is also true for merger of vorticity in a single layer.

References

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