1. Gyenge alak

$$\underline{\underline{\sigma}} \cdot \nabla + \vec{f} = \vec{0} \qquad / \cdot \vec{v} \qquad / \int\limits_{(v)} \dots dv$$

$$-W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \vec{v} \cdot \overset{\downarrow}{\underline{\underline{\sigma}}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$-W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \overset{\downarrow}{\overline{\vec{v} \cdot \underline{\underline{\sigma}}}} \cdot \nabla dv - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$-W\left(\vec{r}, \vec{v}\right) = \int\limits_{(a)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$\underline{\underline{\sigma}} \cdot \vec{n} = \vec{t} \qquad \vec{r} \in a_t$$

$$-W\left(\vec{r}, \vec{v}\right) = \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv + \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \overset{\downarrow}{\vec{v}} \cdot \underline{\underline{\sigma}} \cdot \nabla dv - \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \underline{\underline{\sigma}} \cdot\cdot\, \vec{v} \circ \nabla dv - \int\limits_{(a_t)} \vec{v} \cdot \vec{t} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \underline{\underline{\sigma}} \cdot\cdot\, \underline{\underline{F}}^{-T} \cdot \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla \cdot \underline{\underline{F}} \cdot \underline{\underline{F}}^{-1} dv - \int\limits_{(a_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(v)} \underline{\underline{F}}^{-1} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T} \cdot\cdot\, \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla \cdot \underline{\underline{F}} dv - \int\limits_{(a_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \vec{n} da - \int\limits_{(v)} \vec{v} \cdot \vec{f} dv = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(V)} \underbrace{J \underline{\underline{F}}^{-1} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T}}_{\underline{\underline{S}}} \cdot\cdot\, \underline{\underline{F}}^{T} \cdot \vec{v} \circ \underbrace{\nabla \cdot \underline{\underline{F}}}_{\nabla_0} dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T} \cdot \vec{N} J dA - \int\limits_{(V)} \vec{v} \cdot \underbrace{\vec{f} J}_{\vec{f}_0} dV = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot\cdot\, \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \underbrace{\underline{\underline{F}}^{-1} \cdot J \underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T}}_{\underline{\underline{S}}} \cdot \vec{N} dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot\cdot\, \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \underbrace{\underline{\underline{S}} \cdot \vec{N}}_{\vec{t}_0} dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$W\left(\vec{r}, \vec{v}\right) = \int\limits_{(V)} \underline{\underline{S}} \cdot\cdot\, \underline{\underline{F}}^{T} \cdot \vec{v} \circ \nabla_0 dV - \int\limits_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA - \int\limits_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

2. Nemlineáris rész

$$W(\vec{r},\vec{v}) = \int_{(V)} \underline{\underline{S}} \cdot\cdot \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 dV - \int_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA - \int_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$W(\vec{r},\vec{v}) = \int_{(V)} \underline{\underline{S}} \cdot\cdot \frac{1}{2}\left(\underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 + \nabla_0 \circ \vec{v} \cdot \underline{\underline{F}}\right) dV - \int_{(A_t)} \vec{v} \cdot \underline{\underline{F}} \cdot \vec{t}_0 dA - \int_{(V)} \vec{v} \cdot \vec{f}_0 dV = 0$$

$$\vec{t}_0 \rightarrow \underline{\underline{t}} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\vec{f}_0 \rightarrow \underline{\underline{f}} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$\vec{v} \rightarrow \underline{\underline{v}} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N h_i(\xi,\eta,\zeta)\,\phi^e_{xi} \\ \sum_{i=1}^N h_i(\xi,\eta,\zeta)\,\phi^e_{yi} \\ \sum_{i=1}^N h_i(\xi,\eta,\zeta)\,\phi^e_{zi} \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} h_1 & 0 & 0 & h_2 & 0 & 0 & \cdots & h_N & 0 & 0 \\ 0 & h_1 & 0 & 0 & h_2 & 0 & \cdots & 0 & h_N & 0 \\ 0 & 0 & h_1 & 0 & 0 & h_2 & \cdots & 0 & 0 & h_N \end{bmatrix}}_{\substack{\underline{\underline{H}}(\xi,\eta,\zeta) \\ (3\times 3N)}} \underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\substack{\underline{\underline{\phi}} \\ (3N\times 1)}} = \underline{\underline{H}}(\xi,\eta,\zeta)\,\underline{\underline{\phi}}$$

$$\underline{\underline{H}}(\xi,\eta,\zeta) = \underbrace{\begin{bmatrix} h_1 & 0 & 0 & h_2 & 0 & 0 & \cdots & h_N & 0 & 0 \\ 0 & h_1 & 0 & 0 & h_2 & 0 & \cdots & 0 & h_N & 0 \\ 0 & 0 & h_1 & 0 & 0 & h_2 & \cdots & 0 & 0 & h_N \end{bmatrix}}_{(3\times 3N)}$$

$$\underline{\underline{S}} \rightarrow \underline{\underline{S}} = \begin{bmatrix} S_x & S_{xy} & S_{xz} \\ S_{yx} & S_y & S_{yz} \\ S_{zx} & S_{zy} & S_z \end{bmatrix}$$

Ha $\underline{\underline{S}} = \underline{\underline{S}}^T$

$$\underline{\underline{S}} \rightarrow \underline{\underline{S}} = \begin{bmatrix} S_x & S_{xy} & S_{zx} \\ S_{xy} & S_y & S_{yz} \\ S_{zx} & S_{yz} & S_z \end{bmatrix} \rightarrow \begin{bmatrix} S_x \\ S_y \\ S_z \\ S_{xy} \\ S_{yz} \\ S_{zx} \end{bmatrix}$$

$$W^e(\vec{r},\vec{v}) = \int_{(v)} \left(\underline{\underline{\phi}}^e\right)^T \left(\underline{\underline{B}}^e_L\right)^T \underline{\underline{S}} dv - \int_{(a_t)} \left(\underline{\underline{\phi}}^e\right)^T \underline{\underline{H}}^T \underline{\underline{F}}\underline{\underline{t}} da - \int_{(v)} \left(\underline{\underline{\phi}}^e\right)^T \underline{\underline{H}}^T \underline{\underline{f}} dv = 0$$

$$W^e(\vec{r},\vec{v}) = \left(\underline{\underline{\phi}}^e\right)^T \left[\int_{(v)} \left(\underline{\underline{B}}^e_L\right)^T \underline{\underline{S}} dv - \int_{(a_t)} \underline{\underline{H}}^T \underline{\underline{F}}\underline{\underline{t}} da - \int_{(v)} \underline{\underline{H}}^T \underline{\underline{f}} dv\right] = 0$$

3. Linearizált rész

$$DW\left(\vec{r},\vec{v}\right)\left[\vec{u}\right]=D\left(\int\limits_{(V)}\underline{\underline{S}}\cdot\cdot\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0 dV-\int\limits_{(A_t)}\vec{v}\cdot\underline{\underline{F}}\cdot\vec{t}_0 dA-\int\limits_{(V)}\vec{v}\cdot\vec{f}_0 dV\right)\left[\vec{u}\right]=$$

$$=D\int\limits_{(V)}\underline{\underline{S}}\cdot\cdot\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0 dV\left[\vec{u}\right]-D\int\limits_{(A_t)}\vec{v}\cdot\underline{\underline{F}}\cdot\vec{t}_0 dA\left[\vec{u}\right]-\underbrace{D\int\limits_{(V)}\vec{v}\cdot\vec{f}_0 dV\left[\vec{u}\right]}_{0}=$$

$$=\int\limits_{(V)}D\underline{\underline{S}}\left[\vec{u}\right]\cdot\cdot\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0 dV+\int\limits_{(V)}\underline{\underline{S}}\cdot\cdot D\underline{\underline{F}}^T\left[\vec{u}\right]\cdot\vec{v}\circ\nabla_0 dV+\int\limits_{(A_t)}\vec{v}\cdot D\underline{\underline{F}}\left[\vec{u}\right]\cdot\vec{t}_0 dA$$

$$D\underline{\underline{S}}\left[\vec{u}\right]=\frac{\partial\underline{\underline{S}}}{\partial\underline{\underline{E}}}\cdot\cdot D\underline{\underline{E}}\left[\vec{u}\right]$$

$$D\underline{\underline{E}}\left[\vec{u}\right]=D\left(\frac{1}{2}\left(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{I}}\right)\right)\left[\vec{u}\right]=\frac{1}{2}\left(D\underline{\underline{F}}^T\left[\vec{u}\right]\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot D\underline{\underline{F}}\left[\vec{u}\right]\right)$$

$$D\underline{\underline{F}}\left[\vec{u}\right]=\lim_{\varepsilon\to 0}\frac{d\left(\left(\vec{r}+\varepsilon\vec{u}\right)\circ\nabla_0\right)}{d\varepsilon}=\lim_{\varepsilon\to 0}\frac{d\left(\vec{r}+\varepsilon\vec{u}\right)}{d\varepsilon}\circ\nabla_0=\vec{u}\circ\nabla_0$$

$$D\underline{\underline{F}}^T\left[\vec{u}\right]=\lim_{\varepsilon\to 0}\frac{d\left(\nabla_0\circ\left(\vec{r}+\varepsilon\vec{u}\right)\right)}{d\varepsilon}=\nabla_0\circ\lim_{\varepsilon\to 0}\frac{d\left(\vec{r}+\varepsilon\vec{u}\right)}{d\varepsilon}=\nabla_0\circ\vec{u}$$

$$D\underline{\underline{E}}\left[\vec{u}\right]=D\left(\frac{1}{2}\left(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{I}}\right)\right)\left[\vec{u}\right]=\frac{1}{2}\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)$$

$$D\underline{\underline{S}}\left[\vec{u}\right]=\frac{\partial\underline{\underline{S}}}{\partial\underline{\underline{E}}}\cdot\cdot\frac{1}{2}\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)=2\frac{\partial\underline{\underline{S}}}{\partial\underline{\underline{C}}}\cdot\cdot\frac{1}{2}\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)=$$

$$=\underbrace{\frac{\partial\underline{\underline{S}}}{\partial\underline{\underline{C}}}}_{2\underline{\underline{\mathbb{C}}}^{(4)}}\cdot\cdot\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)$$

$$D\underline{\underline{S}}\left[\vec{u}\right]\cdot\cdot\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0=\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0\cdot\cdot 2\underline{\underline{\mathbb{C}}}^{(4)}\cdot\cdot\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)=$$

$$=\frac{1}{2}\left(\nabla_0\circ\vec{v}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0\right)\cdot\cdot 2\underline{\underline{\mathbb{C}}}^{(4)}\cdot\cdot\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)=$$

$$=\left(\nabla_0\circ\vec{v}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0\right)\cdot\cdot\underline{\underline{\mathbb{C}}}^{(4)}\cdot\cdot\left(\nabla_0\circ\vec{u}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{u}\circ\nabla_0\right)$$

$$\nabla_0\circ\vec{v}\cdot\underline{\underline{F}}+\underline{\underline{F}}^T\cdot\vec{v}\circ\nabla_0=v_{i,j}F_{ik}+F_{ji}v_{i,k}=H_{il,j}\phi_l F_{ik}+F_{ji}H_{il,k}\phi_l=$$

$$=\left(H_{il,j}F_{ik}+F_{ji}H_{il,k}\right)\phi_l$$

$$v_{i,j}F_{ik}+F_{ji}v_{i,k}\to i=x,y,z\qquad\begin{matrix}j=x\\k=x\end{matrix},\quad\begin{matrix}j=y\\k=y\end{matrix},\quad\begin{matrix}j=z\\k=z\end{matrix},\quad\begin{matrix}j=x\\k=y\end{matrix},\quad\begin{matrix}j=y\\k=z\end{matrix},\quad\begin{matrix}j=z\\k=x\end{matrix}$$

$$\vec{v}\circ\nabla\cdot\underline{\underline{F}}=\underbrace{\begin{bmatrix}H_{11,1}F_{11} & H_{22,1}F_{21} & H_{33,1}F_{31} & H_{14,1}F_{11} & H_{25,1}F_{21} & H_{36,1}F_{31} & \cdots\\ H_{11,2}F_{12} & H_{22,2}F_{22} & H_{33,2}F_{32} & H_{14,2}F_{12} & H_{25,2}F_{22} & H_{36,2}F_{32} & \cdots\\ H_{11,3}F_{13} & H_{22,3}F_{23} & H_{33,3}F_{33} & H_{14,3}F_{13} & H_{25,3}F_{23} & H_{36,3}F_{33} & \cdots\\ H_{11,1}F_{12} & H_{22,1}F_{22} & H_{33,1}F_{32} & H_{14,1}F_{12} & H_{25,1}F_{22} & H_{36,1}F_{32} & \cdots\\ H_{11,2}F_{13} & H_{22,2}F_{23} & H_{33,2}F_{33} & H_{14,2}F_{13} & H_{25,2}F_{23} & H_{36,2}F_{33} & \cdots\\ H_{11,3}F_{11} & H_{22,3}F_{21} & H_{33,3}F_{31} & H_{14,3}F_{11} & H_{25,3}F_{21} & H_{36,3}F_{31} & \cdots\end{bmatrix}}_{\underset{(6\times 3N)}{\underline{\underline{B}}^e_{L1}(\xi,\eta,\zeta)}}\underbrace{\begin{bmatrix}\phi^e_{x1}\\ \phi^e_{y1}\\ \phi^e_{z1}\\ \phi^e_{x2}\\ \phi^e_{y2}\\ \phi^e_{z2}\\ \vdots\\ \phi^e_{xN}\\ \phi^e_{yN}\\ \phi^e_{zN}\end{bmatrix}}_{\underset{(3N\times 1)}{\underline{\underline{\phi}}}}=$$

$$
= \underbrace{\begin{bmatrix}
\frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \frac{\partial h_2}{\partial X}F_x & \frac{\partial h_2}{\partial X}F_{yx} & \frac{\partial h_2}{\partial X}F_{zx} & \cdots & \frac{\partial h_N}{\partial X}F_x & \frac{\partial h_N}{\partial X}F_{yx} & \frac{\partial h_N}{\partial X}F_{zx} \\
\frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \frac{\partial h_2}{\partial Y}F_{xy} & \frac{\partial h_2}{\partial Y}F_y & \frac{\partial h_2}{\partial Y}F_{zy} & \cdots & \frac{\partial h_N}{\partial Y}F_{xy} & \frac{\partial h_N}{\partial Y}F_y & \frac{\partial h_N}{\partial Y}F_{zy} \\
\frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \frac{\partial h_2}{\partial Z}F_{xz} & \frac{\partial h_2}{\partial Z}F_{yz} & \frac{\partial h_2}{\partial Z}F_z & \cdots & \frac{\partial h_N}{\partial Z}F_{xz} & \frac{\partial h_N}{\partial Z}F_{yz} & \frac{\partial h_N}{\partial Z}F_z \\
\frac{\partial h_1}{\partial X}F_{xy} & \frac{\partial h_1}{\partial X}F_y & \frac{\partial h_1}{\partial X}F_{zy} & \frac{\partial h_2}{\partial X}F_{xy} & \frac{\partial h_2}{\partial X}F_y & \frac{\partial h_2}{\partial X}F_{zy} & \cdots & \frac{\partial h_N}{\partial X}F_{xy} & \frac{\partial h_N}{\partial X}F_y & \frac{\partial h_N}{\partial X}F_{zy} \\
\frac{\partial h_1}{\partial Y}F_{xz} & \frac{\partial h_1}{\partial Y}F_{yz} & \frac{\partial h_1}{\partial Y}F_z & \frac{\partial h_2}{\partial Y}F_{xz} & \frac{\partial h_2}{\partial Y}F_{yz} & \frac{\partial h_2}{\partial Y}F_z & \cdots & \frac{\partial h_N}{\partial Y}F_{xz} & \frac{\partial h_N}{\partial Y}F_{yz} & \frac{\partial h_N}{\partial Y}F_z \\
\frac{\partial h_1}{\partial Z}F_x & \frac{\partial h_1}{\partial Z}F_{yx} & \frac{\partial h_1}{\partial Z}F_{zx} & \frac{\partial h_2}{\partial Z}F_x & \frac{\partial h_2}{\partial Z}F_{yx} & \frac{\partial h_2}{\partial Z}F_{zx} & \cdots & \frac{\partial h_N}{\partial Z}F_x & \frac{\partial h_N}{\partial Z}F_{yx} & \frac{\partial h_N}{\partial Z}F_{zx}
\end{bmatrix}}_{\underset{(6\times 3N)}{\underline{\underline{B}}^e_{L1}(\xi,\eta,\zeta)}}
\underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underset{(3N\times 1)}{\underline{\underline{\phi}}}}
$$

$$
\vec{v} \circ \nabla \cdot \underline{\underline{F}} = \underbrace{\begin{bmatrix}
H_{11,1}F_{11} & H_{22,1}F_{21} & H_{33,1}F_{31} & H_{14,1}F_{11} & H_{25,1}F_{21} & H_{36,1}F_{31} & \cdots \\
H_{11,2}F_{12} & H_{22,2}F_{22} & H_{33,2}F_{32} & H_{14,2}F_{12} & H_{25,2}F_{22} & H_{36,2}F_{32} & \cdots \\
H_{11,3}F_{13} & H_{22,3}F_{23} & H_{33,3}F_{33} & H_{14,3}F_{13} & H_{25,3}F_{23} & H_{36,3}F_{33} & \cdots \\
H_{11,2}F_{11} & H_{22,2}F_{21} & H_{33,2}F_{31} & H_{14,2}F_{11} & H_{25,2}F_{21} & H_{36,2}F_{31} & \cdots \\
H_{11,3}F_{12} & H_{22,3}F_{22} & H_{33,3}F_{32} & H_{14,3}F_{12} & H_{25,3}F_{22} & H_{36,3}F_{32} & \cdots \\
H_{11,1}F_{13} & H_{22,1}F_{23} & H_{33,1}F_{33} & H_{14,1}F_{13} & H_{25,1}F_{23} & H_{36,1}F_{33} & \cdots
\end{bmatrix}}_{\underset{(6\times 3N)}{\underline{\underline{B}}^e_{L2}(\xi,\eta,\zeta)}}
\underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underset{(3N\times 1)}{\underline{\underline{\phi}}}} =
$$

$$
= \underbrace{\begin{bmatrix}
\frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \frac{\partial h_2}{\partial X}F_x & \frac{\partial h_2}{\partial X}F_{yx} & \frac{\partial h_2}{\partial X}F_{zx} & \cdots & \frac{\partial h_N}{\partial X}F_x & \frac{\partial h_N}{\partial X}F_{yx} & \frac{\partial h_N}{\partial X}F_{zx} \\
\frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \frac{\partial h_2}{\partial Y}F_{xy} & \frac{\partial h_2}{\partial Y}F_y & \frac{\partial h_2}{\partial Y}F_{zy} & \cdots & \frac{\partial h_N}{\partial Y}F_{xy} & \frac{\partial h_N}{\partial Y}F_y & \frac{\partial h_N}{\partial Y}F_{zy} \\
\frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \frac{\partial h_2}{\partial Z}F_{xz} & \frac{\partial h_2}{\partial Z}F_{yz} & \frac{\partial h_2}{\partial Z}F_z & \cdots & \frac{\partial h_N}{\partial Z}F_{xz} & \frac{\partial h_N}{\partial Z}F_{yz} & \frac{\partial h_N}{\partial Z}F_z \\
\frac{\partial h_1}{\partial Y}F_x & \frac{\partial h_1}{\partial Y}F_{yx} & \frac{\partial h_1}{\partial Y}F_{zx} & \frac{\partial h_2}{\partial Y}F_x & \frac{\partial h_2}{\partial Y}F_{yx} & \frac{\partial h_2}{\partial Y}F_{zx} & \cdots & \frac{\partial h_N}{\partial Y}F_x & \frac{\partial h_N}{\partial Y}F_{yx} & \frac{\partial h_N}{\partial Y}F_{zx} \\
\frac{\partial h_1}{\partial Z}F_{xy} & \frac{\partial h_1}{\partial Z}F_y & \frac{\partial h_1}{\partial Z}F_{zy} & \frac{\partial h_2}{\partial Z}F_{xy} & \frac{\partial h_2}{\partial Z}F_y & \frac{\partial h_2}{\partial Z}F_{zy} & \cdots & \frac{\partial h_N}{\partial Z}F_{xy} & \frac{\partial h_N}{\partial Z}F_y & \frac{\partial h_N}{\partial Z}F_{zy} \\
\frac{\partial h_1}{\partial X}F_{xz} & \frac{\partial h_1}{\partial X}F_{yz} & \frac{\partial h_1}{\partial X}F_z & \frac{\partial h_2}{\partial X}F_{xz} & \frac{\partial h_2}{\partial X}F_{yz} & \frac{\partial h_2}{\partial X}F_z & \cdots & \frac{\partial h_N}{\partial X}F_{xz} & \frac{\partial h_N}{\partial X}F_{yz} & \frac{\partial h_N}{\partial X}F_z
\end{bmatrix}}_{\underset{(6\times 3N)}{\underline{\underline{B}}^e_{L2}(\xi,\eta,\zeta)}}
\underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underset{(3N\times 1)}{\underline{\underline{\phi}}}}
$$

$$
\nabla_0 \circ \vec{v} \cdot \underline{\underline{F}} + \underline{\underline{F}}^T \cdot \vec{v} \circ \nabla_0 =
$$

$$
= \underbrace{\begin{bmatrix}
\frac{\partial h_1}{\partial X}F_x & \frac{\partial h_1}{\partial X}F_{yx} & \frac{\partial h_1}{\partial X}F_{zx} & \cdots \\
\frac{\partial h_1}{\partial Y}F_{xy} & \frac{\partial h_1}{\partial Y}F_y & \frac{\partial h_1}{\partial Y}F_{zy} & \cdots \\
\frac{\partial h_1}{\partial Z}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yz} & \frac{\partial h_1}{\partial Z}F_z & \cdots \\
\frac{\partial h_1}{\partial X}F_{xy} + \frac{\partial h_1}{\partial Y}F_x & \frac{\partial h_1}{\partial X}F_y + \frac{\partial h_1}{\partial Y}F_{yx} & \frac{\partial h_1}{\partial X}F_{zy} + \frac{\partial h_1}{\partial Y}F_{zx} & \cdots \\
\frac{\partial h_1}{\partial Y}F_{xz} + \frac{\partial h_1}{\partial Z}F_{xy} & \frac{\partial h_1}{\partial Y}F_{yz} + \frac{\partial h_1}{\partial Z}F_y & \frac{\partial h_1}{\partial Y}F_z + \frac{\partial h_1}{\partial Z}F_{zy} & \cdots \\
\frac{\partial h_1}{\partial Z}F_x + \frac{\partial h_1}{\partial X}F_{xz} & \frac{\partial h_1}{\partial Z}F_{yx} + \frac{\partial h_1}{\partial X}F_{yz} & \frac{\partial h_1}{\partial Z}F_{zx} + \frac{\partial h_1}{\partial X}F_z & \cdots
\end{bmatrix}}_{\underset{(6\times 3N)}{\underline{\underline{B}}^e_{L}(\xi,\eta,\zeta)}}
\underbrace{\begin{bmatrix} \phi^e_{x1} \\ \phi^e_{y1} \\ \phi^e_{z1} \\ \phi^e_{x2} \\ \phi^e_{y2} \\ \phi^e_{z2} \\ \vdots \\ \phi^e_{xN} \\ \phi^e_{yN} \\ \phi^e_{zN} \end{bmatrix}}_{\underset{(3N\times 1)}{\underline{\underline{\phi}}}}
$$

$$\underline{\underline{S}} \cdot\cdot D\underline{\underline{F}}^T[\vec{u}] \cdot \vec{v} \circ \nabla_0 = \underline{\underline{S}} \cdot\cdot \frac{1}{2}\left(D\underline{\underline{F}}^T[\vec{u}] \cdot \vec{v} \circ \nabla_0 + \nabla_0 \circ \vec{v} \cdot D\underline{\underline{F}}[\vec{u}]\right) =$$

$$= \underline{\underline{S}} \cdot\cdot \frac{1}{2}\left(\nabla_0 \circ \vec{u} \cdot \vec{v} \circ \nabla_0 + \nabla_0 \circ \vec{v} \cdot \vec{u} \circ \nabla_0\right) =$$