

How to read the tables

Below a short overview over the different colums is given. To use the quadrature rules only the last two rows (ShunnHam & Reference) are of importance. As these two columns contain all the information needed to build the quadrature method. The column 'ShunnHam' indicates how to combine simplex quadrature rule into the correct Duffy transformation. The column 'Reference' gives the location of quadrature points on the reference element. Here the reference Tetrahedron is: (0,0,0), (1,0,0), (0,1,0), (0,0,1) and the reference triangle is the: (0,0), (1,0), (0,1).

The table also contain more information that was used for the identification of the correct Duffy transformation after the relative coordiantes transformation.

No.	Internal numbering of the subdomain. If there are two numbers there, this means that the two domains have the same shape and can be easily tranformed into each other. For the affected tables the transformation is given above the table.
Domain.	Integration bounds of the subdomain after relative coordinate transformation.
Corners	Corners of the subdomain domain in (z,x)-coordinates.
Idx	As it is difficult to find transformation from a refrence subdomain to the actual subdomain by hand, a brute froce algrithm was use to find a good mapping and this number in dicates which mapping was chossen.
Mapping	Mapping from the reference subdomain for which one can find a suitable Duffy transformation quite easily to the actual subdo-main. Attention: The coordinates reverse to (x,z) instead of (z,x).
(x,z)	ξ_i resp. η_i are points on the reference subdomain transformed by the mapping in the previous column.
(x,y)	Transformed points on in the original coordinates (x,y). Form the Duffy transomation the shape of the refrence subdomain can be deduced.
Tensor-Product	Duffy transformation to the hypercube.
ShunnHam	Duffy transformation of the more economical simplex tensor-product.
Reference	Coordinates of the quadrature points on the reference elements.

Triangle-Triangle

Positive Distance

No.	Domain	(x,y)	Tensor-Prodcut	ShunnHamm	Reference
1	$0 \leq x_1 \leq 1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
	$0 \leq x_2 \leq x_1$	$x_2 = \xi_2$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq y_1 \leq 1$	$y_1 = \xi_3$	$\xi_3 = x_3$	$\xi_3 = y_1$	$v = \begin{bmatrix} 1 - \xi_3 \\ \xi_4 \end{bmatrix}$
	$0 \leq y_2 \leq y_1$	$y_2 = \xi_4$	$\xi_4 = x_3 x_4$	$\xi_4 = y_2$	

Common Vertex

Relative coordinate transformation: $\vec{z} = (x_1, x_2, y_1, y_2)$

No.	Domain	(x,y)	Tensor-Prodcut	ShunnHamm	Reference
1	$0 \leq z_1 \leq 1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$	$x_2 = \xi_2$	$\xi_2 = x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq z_1$	$y_1 = \xi_3$	$\xi_3 = x_1 x_3$	$\xi_3 = x_1 y_1$	$v = \begin{bmatrix} 1 - \xi_3 \\ \xi_4 \end{bmatrix}$
	$0 \leq z_4 \leq z_3$	$y_2 = \xi_4$	$\xi_4 = x_1 x_3 x_4$	$\xi_4 = x_1 y_2$	
2	$0 \leq z_3 \leq 1$	$x_1 = \xi_3$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_3 \\ \xi_4 \end{bmatrix}$
	$0 \leq z_4 \leq z_3$	$x_2 = \xi_4$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_1 \leq z_3$	$y_1 = \xi_1$	$\xi_3 = x_1 x_3$	$\xi_3 = x_1 y_1$	$v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$	$y_2 = \xi_2$	$\xi_4 = x_1 x_3 x_4$	$\xi_4 = x_1 y_2$	

Common Edge

Relative coordinate transformation: $(x_1, z_1, z_2, z_3) = (x_1, y_1 - x_1, y_2, x_2)$

No.	Domain	Corners	Idx	Mapping	(x, z)	(x, y)	Tensor-Product	ShunnHam	Reference
1	$-1 \leq z_1 \leq 0$ $0 \leq z_2 \leq 1 + z_1$ $0 \leq z_3 \leq z_2 - z_1$ $z_2 - z_1 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	63	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$x_1 = \xi_1$ $z_1 = -\xi_3$ $z_2 = \xi_2 - \xi_3$ $z_3 = \xi_4$	$x_1 = \xi_1$ $x_2 = \xi_4$ $y_1 = \xi_1 - \xi_3$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\xi_4 = x_2 z_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3) \\ \xi_2 - \xi_3 \end{bmatrix}$
2	$-1 \leq z_1 \leq 0$ $0 \leq z_2 \leq 1 + z_1$ $z_2 - z_1 \leq z_3 \leq 1$ $z_3 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	10	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $z_1 = -\xi_4$ $z_2 = \xi_3 - \xi_4$ $z_3 = \xi_2$	$x_1 = \xi_1$ $x_2 = \xi_2$ $y_1 = \xi_1 - \xi_4$ $y_2 = \xi_3 - \xi_4$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\xi_4 = x_2 y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_3 - \xi_4 \end{bmatrix}$
3	$0 \leq z_1 \leq 1$ $0 \leq z_2 \leq z_1$ $0 \leq z_3 \leq 1 - z_1$ $z_3 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_3$ $z_1 = \xi_3$ $z_2 = \xi_4$ $z_3 = \xi_2 - \xi_3$	$x_1 = \xi_1 - \xi_3$ $x_2 = \xi_2 - \xi_3$ $y_1 = \xi_1$ $y_2 = \xi_4$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\xi_4 = x_2 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_3) \\ \xi_2 - \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_4 \end{bmatrix}$
4	$0 \leq z_1 \leq 1$ $z_1 \leq z_2 \leq 1$ $0 \leq z_3 \leq z_2 - z_1$ $z_2 - z_1 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4$ $z_1 = \xi_4$ $z_2 = \xi_2$ $z_3 = \xi_3 - \xi_4$	$x_1 = \xi_1 - \xi_4$ $x_2 = \xi_3 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_2$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\xi_4 = x_2 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
5	$0 \leq z_1 \leq 1$ $z_1 \leq z_2 \leq 1$ $z_2 - z_1 \leq z_3 \leq 1 - z_1$ $z_3 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4$ $z_1 = \xi_4$ $z_2 = \xi_3$ $z_3 = \xi_2 - \xi_4$	$x_1 = \xi_1 - \xi_4$ $x_2 = \xi_2 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\xi_4 = x_2 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_2 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_3 \end{bmatrix}$

Common Face

Relative coordinate transformation: $(x_1, x_2, z_1, z_2) = (x_1, x_2, y_1 - x_1, y_2 - x_2)$

Reference A: $(\hat{x}, \hat{z}) = (x, -z)$, Reference B: $(\hat{x}, \hat{z}) = (x + z, z)$

No.	Domain	Corners	Idx	Mapping	(\hat{x}, \hat{z})	(x, y)	Tensor-Product	ShunnHam	Reference A	Reference B
1/6	$0 \leq \hat{z}_1 \leq 1$ $\hat{z}_1 \leq \hat{z}_2 \leq 1$ $\hat{z}_2 \leq \hat{x}_1 \leq 1$ $\hat{z}_2 \leq \hat{x}_2 \leq \hat{x}_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	7	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\hat{x}_1 = \xi_1$ $\hat{x}_2 = \xi_1 - \xi_2 + \xi_3$ $\hat{z}_1 = \xi_4$ $\hat{z}_2 = \xi_3$	$x_1 = \xi_1$ $x_2 = \xi_1 - \xi_2 + \xi_3$ $y_1 = \xi_1 - \xi_4$ $y_2 = \xi_1 - \xi_2$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_1 - \xi_2 \end{bmatrix}$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_1 - \xi_2 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \xi_3 \end{bmatrix}$
2/5	$0 \leq \hat{z}_1 \leq 1$ $0 \leq \hat{z}_2 \leq \hat{z}_1$ $\hat{z}_1 \leq \hat{x}_1 \leq 1$ $\hat{z}_2 \leq \hat{x}_2 \leq \hat{x}_1 - \hat{z}_1 + \hat{z}_2$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\hat{x}_1 = \xi_1$ $\hat{x}_2 = \xi_2 - \xi_3 + \xi_4$ $\hat{z}_1 = \xi_2$ $\hat{z}_2 = \xi_4$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_3 + \xi_4$ $y_1 = \xi_1 - \xi_3$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 + \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3) \\ \xi_2 - \xi_3 \end{bmatrix}$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_3) \\ \xi_2 - \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 + \xi_4 \end{bmatrix}$
3/4	$-1 \leq \hat{z}_1 \leq 0$ $0 \leq \hat{z}_2 \leq 1 + \hat{z}_1$ $\hat{z}_2 \leq \hat{x}_1 \leq 1 + \hat{z}_1$ $\hat{z}_2 \leq \hat{x}_2 \leq \hat{x}_1$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	10	$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$	$\hat{x}_1 = \xi_1 - \xi_4$ $\hat{x}_2 = \xi_2 - \xi_4$ $\hat{z}_1 = -\xi_4$ $\hat{z}_2 = \xi_3 - \xi_4$	$x_1 = \xi_1 - \xi_4$ $x_2 = \xi_2 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 x_4$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_2 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 \end{bmatrix}$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_2 - \xi_4 \end{bmatrix}$

Tetrahedron-Triangle

Positive Distance

No.	Domain	(x, y)	Tensor-Product	ShunnHam	Reference
1	$0 \leq x_1 \leq 1$ $0 \leq x_2 \leq x_1$ $0 \leq x_3 \leq x_2$ $0 \leq y_1 \leq 1$ $0 \leq y_2 \leq y_1$	$x_1 = \xi_1$ $x_2 = \xi_2$ $x_3 = \xi_3$ $y_1 = \xi_4$ $y_2 = \xi_5$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_4$ $\xi_5 = x_4 x_5$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = y_1$ $\xi_5 = y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_4 \\ \xi_5 \end{bmatrix}$

Common Vertex

Relative coordinate transformation: $\vec{z} = (x_1, x_2, x_3, y_1, y_2)$

No.	Domain	(x, y)	Tensor-Product	ShunnHam	Reference
1	$0 \leq z_1 \leq 1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_4 \\ \xi_5 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$	$x_2 = \xi_2$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq z_2$	$x_3 = \xi_3$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_3$	
	$0 \leq z_4 \leq z_1$	$y_1 = \xi_4$	$\xi_4 = x_1 x_4$	$\xi_4 = x_1 y_1$	
	$0 \leq z_5 \leq z_4$	$y_2 = \xi_5$	$\xi_5 = x_1 x_4 x_5$	$\xi_5 = x_1 y_2$	
2	$0 \leq z_4 \leq 1$	$x_1 = \xi_3$	$\xi_1 = x_1$	$\xi_1 = y_1$	$u = \begin{bmatrix} 1 - \xi_3 \\ \xi_3 - \xi_4 \\ \xi_5 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
	$0 \leq z_5 \leq z_4$	$x_2 = \xi_4$	$\xi_2 = x_1 x_2$	$\xi_2 = y_2$	
	$0 \leq z_1 \leq z_4$	$x_3 = \xi_5$	$\xi_3 = x_1 x_3$	$\xi_3 = y_1 x_1$	
	$0 \leq z_2 \leq z_1$	$y_1 = \xi_1$	$\xi_4 = x_1 x_3 x_4$	$\xi_4 = y_1 x_2$	
	$0 \leq z_3 \leq z_2$	$y_2 = \xi_2$	$\xi_5 = x_1 x_3 x_4 x_5$	$\xi_5 = y_1 x_3$	

Common Edge

Relative coordinate transformation: $(x_1, z_1, z_2, z_3, z_4) = (x_1, y_1 - x_1, y_2, x_2, x_3)$

No.	Domain	Corners	Idx	Mapping	(x, z)	(x, y)	Tensor-Product	ShunnHam	Reference
1	$-1 \leq z_1 \leq 0$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	6	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$	$x_1 = \xi_1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_3 \\ \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_5) \\ \xi_2 - \xi_5 \end{bmatrix}$
	$0 \leq z_2 \leq 1 + z_1$				$z_1 = -\xi_5$	$x_2 = \xi_3$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq z_2 - z_1$				$z_2 = \xi_2 - \xi_5$	$x_3 = \xi_4$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_2 y_1$	
	$0 \leq z_4 \leq z_3$				$z_3 = \xi_3$	$y_1 = \xi_1 - \xi_5$	$\xi_4 = x_1 x_2 x_3 y_1$	$\xi_4 = x_2 y_2$	
	$z_2 - z_1 \leq x_1 \leq 1$				$z_4 = \xi_4$	$y_2 = \xi_2 - \xi_5$	$\xi_5 = x_1 x_2 y_2$	$\xi_5 = x_2 z_1$	
2	$-1 \leq z_1 \leq 0$	$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	42	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$x_1 = \xi_1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_5 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_3 - \xi_4 \end{bmatrix}$
	$0 \leq z_2 \leq 1 + z_1$				$z_1 = -\xi_4$	$x_2 = \xi_2$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$z_2 - z_1 \leq z_3 \leq 1$				$z_2 = \xi_3 - \xi_4$	$x_3 = \xi_5$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_2 y_1$	
	$0 \leq z_4 \leq z_3$				$z_3 = \xi_2$	$y_1 = \xi_1 - \xi_4$	$\xi_4 = x_1 x_2 x_3 y_1$	$\xi_4 = x_2 y_2$	
	$z_3 \leq x_1 \leq 1$				$z_4 = \xi_5$	$y_2 = \xi_3 - \xi_4$	$\xi_5 = x_1 x_2 y_2$	$\xi_5 = x_2 z_1$	
3	$0 \leq z_1 \leq 1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4$	$x_1 = \xi_1 - \xi_4$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_1 - \xi_2 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_5 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$				$z_1 = \xi_4$	$x_2 = \xi_2 - \xi_4$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq 1 - z_1$				$z_2 = \xi_5$	$x_3 = \xi_3 - \xi_4$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_2 y_1$	
	$0 \leq z_4 \leq z_3$				$z_3 = \xi_2 - \xi_4$	$y_1 = \xi_1$	$\xi_4 = x_1 x_2 x_3 y_1$	$\xi_4 = x_2 y_2$	
	$z_3 \leq x_1 \leq 1 - z_1$				$z_4 = \xi_3 - \xi_4$	$y_2 = \xi_5$	$\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_5 = x_2 z_3$	
4	$0 \leq z_1 \leq 1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$	2	$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$	$x_1 = \xi_1 - \xi_5$	$x_1 = \xi_1 - \xi_5$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_5) \\ \xi_1 - \xi_3 \\ \xi_4 - \xi_5 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$
	$z_1 \leq z_2 \leq 1$				$z_1 = \xi_5$	$x_2 = \xi_3 - \xi_5$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq z_2 - z_1$				$z_2 = \xi_2$	$x_3 = \xi_4 - \xi_5$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_2 y_1$	
	$0 \leq z_4 \leq z_3$				$z_3 = \xi_3 - \xi_5$	$y_1 = \xi_1$	$\xi_4 = x_1 x_2 x_3 y_1$	$\xi_4 = x_2 y_2$	
	$z_2 - z_1 \leq x_1 \leq 1 - z_1$				$z_4 = \xi_4 - \xi_5$	$y_2 = \xi_2$	$\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_5 = x_2 z_3$	
5	$0 \leq z_1 \leq 1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	98	$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_2 + \xi_3$	$x_1 = \xi_1 - \xi_2 + \xi_3$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_2 + \xi_3) \\ \xi_1 - \xi_2 \\ \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 + \xi_5 \end{bmatrix}$
	$z_1 \leq z_2 \leq 1$				$z_1 = \xi_2 - \xi_3$	$x_2 = \xi_3$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$z_2 - z_1 \leq z_3 \leq 1 - z_1$				$z_2 = \xi_2 - \xi_3 + \xi_5$	$x_3 = \xi_4$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_2 y_1$	
	$0 \leq z_4 \leq z_3$				$z_3 = \xi_3$	$y_1 = \xi_1$	$\xi_4 = x_1 x_2 x_3 y_1$	$\xi_4 = x_2 y_2$	
	$z_3 \leq x_1 \leq 1 - z_1$				$z_4 = \xi_4$	$y_2 = \xi_2 - \xi_3 + \xi_5$	$\xi_5 = x_1 x_2 x_3 y_2$	$\xi_5 = x_2 y_1 z_1$	

Common Face

Relative coordinate transformation: $(x_1, x_2, z_1, z_2, z_3) = (x_1, x_2, y_1 - x_1, y_2 - x_2, x_3)$

No.	Domain	Corners	Idx	Mapping	(x, z)	(x, y)	Tensor-Product	ShunnHam	Reference
1	$-1 \leq z_1 \leq 0$ $-1 \leq z_2 \leq z_1$ $0 \leq z_3 \leq -z_2$ $-z_2 \leq x_2 \leq 1$ $x_2 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$	581	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $x_2 = \xi_2$ $z_1 = -\xi_3 + \xi_5$ $z_2 = -\xi_3$ $z_3 = \xi_3 - \xi_4$	$x_1 = \xi_1$ $x_2 = \xi_2$ $x_3 = \xi_3 - \xi_4$ $y_1 = \xi_1 - \xi_3 + \xi_5$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 z_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3 + \xi_5) \\ \xi_2 - \xi_3 \end{bmatrix}$
2	$-1 \leq z_1 \leq 0$ $-1 \leq z_2 \leq z_1$ $-z_2 \leq z_3 \leq 1$ $z_3 \leq x_2 \leq 1$ $x_2 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	62	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $x_2 = \xi_2$ $z_1 = -\xi_5$ $z_2 = -\xi_3 + \xi_4 - \xi_5$ $z_3 = \xi_3$	$x_1 = \xi_1$ $x_2 = \xi_2$ $x_3 = \xi_3$ $y_1 = \xi_1 - \xi_5$ $y_2 = \xi_2 - \xi_3 + \xi_4 - \xi_5$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_5) \\ \xi_2 - \xi_3 + \xi_4 - \xi_5 \end{bmatrix}$
3	$-1 \leq z_1 \leq 0$ $z_1 \leq z_2 \leq 0$ $0 \leq z_3 \leq -z_2$ $-z_2 \leq x_2 \leq 1 + z_1 - z_2$ $x_2 + z_2 - z_1 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$	91	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_4 + \xi_5$ $z_1 = -\xi_3$ $z_2 = -\xi_3 + \xi_4 - \xi_5$ $z_3 = \xi_3 + \xi_4$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_4 + \xi_5$ $x_3 = \xi_3 - \xi_4$ $y_1 = \xi_1 - \xi_3$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \xi_4 - \xi_5 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3) \\ \xi_2 - \xi_3 \end{bmatrix}$
4	$-1 \leq z_1 \leq 0$ $z_1 \leq z_2 \leq 0$ $-z_2 \leq z_3 \leq 1 + z_1 - z_2$ $z_3 \leq x_2 \leq 1 + z_1 - z_2$ $x_2 + z_2 - z_1 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	61	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_3 + \xi_4$ $z_1 = -\xi_3 + \xi_5$ $z_2 = -\xi_4 + \xi_5$ $z_3 = \xi_4$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_3 + \xi_4$ $x_3 = \xi_4$ $y_1 = \xi_1 - \xi_3 + \xi_5$ $y_2 = \xi_2 - \xi_3 + \xi_5$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 \\ \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3 + \xi_5) \\ \xi_2 - \xi_3 + \xi_5 \end{bmatrix}$
5	$0 \leq z_1 \leq 1$ $z_1 - 1 \leq z_2 \leq 0$ $0 \leq z_3 - z_2$ $-z_2 \leq x_2 \leq 1 - z_1$ $x_2 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	61	$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_5$ $x_2 = \xi_2 - \xi_5$ $z_1 = \xi_5$ $z_2 = -\xi_3 + \xi_5$ $z_3 = \xi_3 - \xi_4$	$x_1 = \xi_1 - \xi_5$ $x_2 = \xi_2 - \xi_5$ $x_3 = \xi_3 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_2 - \xi_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_5) \\ \xi_1 - \xi_2 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 \end{bmatrix}$
6	$0 \leq z_1 \leq 1$ $z_1 - 1 \leq z_2 \leq 0$ $-z_2 \leq z_3 \leq 1 - z_1$ $z_3 \leq x_2 \leq 1 - z_1$ $x_2 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	32	$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4 + \xi_5$ $x_2 = \xi_2 - \xi_4 + \xi_5$ $z_1 = \xi_4 - \xi_5$ $z_2 = -\xi_3 + \xi_4$ $z_3 = \xi_3 - \xi_4 + \xi_5$	$x_1 = \xi_1 - \xi_4 + \xi_5$ $x_2 = \xi_2 - \xi_4 + \xi_5$ $x_3 = \xi_3 - \xi_4 + \xi_5$ $y_1 = \xi_1$ $y_2 = \xi_2 - \xi_3 + \xi_5$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4 + \xi_5) \\ \xi_1 - \xi_2 \\ \xi_3 - \xi_4 + \xi_5 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_3 + \xi_5 \end{bmatrix}$
7	$0 \leq z_1 \leq 1$ $0 \leq z_2 \leq z_1$ $0 \leq z_3 \leq 1 - z_1$ $z_3 \leq x_2 \leq 1 - z_1$ $x_2 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4$ $x_2 = \xi_2 - \xi_4$ $z_1 = \xi_4$ $z_2 = \xi_5$ $z_3 = \xi_3 - \xi_4$	$x_1 = \xi_1 - \xi_4$ $x_2 = \xi_2 - \xi_4$ $x_3 = \xi_3 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_2 - \xi_4 + \xi_5$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 - \xi_4) \\ \xi_1 - \xi_2 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 - \xi_4 + \xi_5 \end{bmatrix}$
8	$-1 \leq z_1 \leq 0$ $0 \leq z_2 \leq 1 + z_1$ $0 \leq z_3 \leq 1 + z_1 - z_2$ $z_3 \leq x_2 \leq 1 + z_1 - z_2$ $x_2 + z_2 - z_1 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$	31	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_3 + \xi_4 - \xi_5$ $z_1 = -\xi_3 + \xi_4$ $z_2 = \xi_5$ $z_3 = \xi_4 - \xi_5$	$x_1 = \xi_1$ $x_2 = \xi_2 - \xi_3 + \xi_4 - \xi_5$ $x_3 = \xi_4 - \xi_5$ $y_1 = \xi_1 - \xi_3 + \xi_4$ $y_2 = \xi_2 - \xi_3 + \xi_4$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 \\ \xi_4 - \xi_5 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_3 + \xi_4) \\ \xi_2 - \xi_3 + \xi_4 \end{bmatrix}$
9	$0 \leq z_1 \leq 1$ $z_1 \leq z_2 \leq 1$ $0 \leq z_3 \leq 1 - z_2$ $z_3 \leq x_2 \leq 1 - z_2$ $x_2 + z_2 - z_1 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	2	$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \xi_4 + \xi_5$ $x_2 = \xi_2 - \xi_4$ $z_1 = \xi_4 - \xi_5$ $z_2 = \xi_4$ $z_3 = \xi_3 - \xi_4$	$x_1 = \xi_1 - \xi_4 + \xi_5$ $x_2 = \xi_2 - \xi_4$ $x_3 = \xi_3 - \xi_4$ $y_1 = \xi_1$ $y_2 = \xi_2$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\xi_4 = x_1 x_2 x_3 y_1$ $\xi_5 = x_1 x_2 x_3 y_1 y_2$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_3$ $\xi_4 = x_3 y_1$ $\xi_5 = x_3 y_2$	$u = \begin{bmatrix} 1 - (\xi_1 + \xi_4 + \xi_5) \\ \xi_1 - \xi_2 + \xi_5 \\ \xi_3 - \xi_4 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_2 \end{bmatrix}$

Tetrahedron-Tetrahedron

Positive Distance

No.	Domain	(x, y)	Tensor-Product	ShunnHam	Reference
1	$0 \leq x_1 \leq 1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$
	$0 \leq x_2 \leq x_1$	$x_2 = \xi_2$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq x_3 \leq x_2$	$x_3 = \xi_3$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_3$	
	$0 \leq y_1 \leq 1$	$y_1 = \xi_4$	$\xi_4 = x_4$	$\xi_4 = y_1$	$v = \begin{bmatrix} 1 - \xi_4 \\ \xi_4 - \xi_5 \\ \xi_6 \end{bmatrix}$
	$0 \leq y_2 \leq y_1$	$y_2 = \xi_5$	$\xi_5 = x_4 x_5$	$\xi_5 = y_2$	
	$0 \leq y_3 \leq y_2$	$y_3 = \xi_6$	$\xi_6 = x_4 x_5 x_6$	$\xi_6 = y_3$	

Common Vertex

Relative coordinate transformation: $\vec{z} = (x_1, x_2, x_3, y_1, y_2, y_3)$

No.	Domain	(x, y)	Tensor-Product	ShunnHam	Reference
1	$0 \leq z_1 \leq 1$	$x_1 = \xi_1$	$\xi_1 = x_1$	$\xi_1 = x_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$	$x_2 = \xi_2$	$\xi_2 = x_1 x_2$	$\xi_2 = x_2$	
	$0 \leq z_3 \leq z_2$	$x_3 = \xi_3$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = x_3$	
	$0 \leq z_4 \leq z_1$	$y_1 = \xi_4$	$\xi_4 = x_1 x_4$	$\xi_4 = x_1 y_1$	$v = \begin{bmatrix} 1 - \xi_4 \\ \xi_4 - \xi_5 \\ \xi_6 \end{bmatrix}$
	$0 \leq z_5 \leq z_4$	$y_2 = \xi_5$	$\xi_5 = x_1 x_4 x_5$	$\xi_5 = x_1 y_2$	
	$0 \leq z_6 \leq z_5$	$y_3 = \xi_6$	$\xi_6 = x_1 x_4 x_5 x_6$	$\xi_6 = x_1 y_3$	
2	$0 \leq z_4 \leq 1$	$x_1 = \xi_4$	$\xi_1 = x_1$	$\xi_1 = y_1$	$u = \begin{bmatrix} 1 - \xi_3 \\ \xi_3 - \xi_4 \\ \xi_5 \end{bmatrix}$
	$0 \leq z_5 \leq z_4$	$x_2 = \xi_5$	$\xi_2 = x_1 x_2$	$\xi_2 = y_2$	
	$0 \leq z_6 \leq z_5$	$x_3 = \xi_6$	$\xi_3 = x_1 x_2 x_3$	$\xi_3 = y_3$	
	$0 \leq z_1 \leq z_4$	$y_1 = \xi_1$	$\xi_4 = x_1 x_4$	$\xi_4 = y_1 x_1$	$v = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_3 \end{bmatrix}$
	$0 \leq z_2 \leq z_1$	$y_2 = \xi_2$	$\xi_5 = x_1 x_4 x_5$	$\xi_5 = y_1 x_2$	
	$0 \leq z_3 \leq z_2$	$y_3 = \xi_3$	$\xi_6 = x_1 x_4 x_5 x_6$	$\xi_6 = y_1 x_3$	

Common Edge

$(x, z) = (x_1, y_1 - x_1, y_2, y_3, x_2, x_3)$

No.	Domain	Corners	Idx	Mapping	(x, z)	(x, y)	Tensor-Product	ShunnHam	Reference
1	$-1 \leq z_1 \leq 0$ $0 \leq z_2 \leq 1 + z_1$ $0 \leq z_3 \leq z_2$ $0 \leq z_4 \leq z_2 - z_1$ $0 \leq z_5 \leq z_4$ $z_2 - z_1 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	1089238	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$	$x_1 = \xi_1$ $z_1 = -\xi_2 + \eta_2$ $z_2 = \eta_2$ $z_3 = \eta_2 - \eta_3$ $z_4 = \xi_2 - \eta_1$ $z_5 = \xi_3 - \eta_1$	$x_1 = \xi_1$ $x_2 = \xi_2 - \eta_1$ $x_3 = \xi_3 - \eta_1$ $y_1 = \xi_1 - \xi_2 + \eta_2$ $y_2 = \eta_2$ $y_3 = \eta_2 - \eta_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\eta_1 = x_1 x_2 x_3 y_1$ $\eta_2 = x_1 x_2 y_2$ $\eta_3 = x_1 x_2 y_2 y_3$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\eta_1 = x_2 y_2$ $\eta_2 = x_2 z_1$ $\eta_3 = x_2 z_2$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 + \eta_1 \\ \xi_3 - \eta_1 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_2 + \eta_2) \\ \xi_1 - \xi_2 \\ \eta_2 - \eta_3 \end{bmatrix}$
2	$-1 \leq z_1 \leq 0$ $0 \leq z_2 \leq 1 + z_1$ $0 \leq z_3 \leq z_2$ $z_2 - z_1 \leq z_4 \leq 1$ $0 \leq z_5 \leq z_4$ $z_4 \leq x_1 \leq 1$	$\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	82555	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$	$x_1 = \xi_1$ $z_1 = -\xi_2 + \xi_3$ $z_2 = \eta_1$ $z_3 = \eta_2$ $z_4 = \xi_2$ $z_5 = \xi_2 - \eta_3$	$x_1 = \xi_1$ $x_2 = \xi_2$ $x_3 = \xi_2 - \eta_3$ $y_1 = \xi_1 - \xi_2 + \xi_3$ $y_2 = \eta_1$ $y_3 = \eta_2$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\eta_1 = x_1 x_2 x_3 y_1$ $\eta_2 = x_1 x_2 x_3 y_1 y_2$ $\eta_3 = x_1 x_2 y_3$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\eta_1 = x_2 y_2$ $\eta_2 = x_2 y_3$ $\eta_3 = x_2 z_1$	$u = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \xi_2 \\ \xi_2 - \eta_3 \end{bmatrix}$ $v = \begin{bmatrix} 1 - (\xi_1 - \xi_2 + \xi_3) \\ \xi_1 - \xi_2 + \xi_3 - eta_1 \\ \eta_2 \end{bmatrix}$
3	$0 \leq z_1 \leq 1$ $0 \leq z_2 \leq z_1$ $0 \leq z_3 \leq z_2$ $0 \leq z_4 \leq 1 - z_1$ $0 \leq z_5 \leq z_4$ $z_4 \leq x_1 \leq 1 - z_1$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	1	$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$	$x_1 = \xi_1 - \eta_1$ $z_1 = \eta_1$ $z_2 = \eta_2$ $z_3 = \eta_3$ $z_4 = \xi_2 - \eta_1$ $z_5 = \xi_3 - \eta_1$	$x_1 = \xi_1 - \eta_1$ $x_2 = \xi_2 - \eta_1$ $x_3 = \xi_3 - \eta_1$ $y_1 = \xi_1$ $y_2 = \eta_2$ $y_3 = \eta_3$	$\xi_1 = x_1$ $\xi_2 = x_1 x_2$ $\xi_3 = x_1 x_2 x_3$ $\eta_1 = x_1 x_2 x_3 y_1$ $\eta_2 = x_1 x_2 x_3 y_1 y_2$ $\eta_3 = x_1 x_2 x_3 y_1 y_2 y_3$	$\xi_1 = x_1$ $\xi_2 = x_2$ $\xi_3 = x_2 y_1$ $\eta_1 = x_2 y_2$ $\eta_2 = x_2 y_3$ $\eta_3 = x_2 y_4$	$u = \begin{bmatrix} 1 - (\xi_1 - \eta_1) \\ \xi_1 - \xi_2 \\ \xi_3 - \eta_1 \end{bmatrix}$ $v = \begin{bmatrix} 1 - \xi_1 \\ \xi_1 - \eta_2 \\ \eta_3 \end{bmatrix}$

