

# Package ‘DER’

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**Type** Package

**Title** Income Polarization Index

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**Maintainer** Michail Tsagris <mtsagris@uoc.gr>

**Depends** R (>= 4.0)

**Imports** rangen, Rfast, Rfast2, Rcpp, RcppParallel, stats

**LinkingTo** Rcpp, RcppParallel

**Encoding** UTF-8

**SystemRequirements** GNU make

**Description** Extremely fast and memory efficient computation of the DER (or PaF) income polarization index as proposed by Duclos J. Y., Esteban, J. and Ray D. (2004). ``Polarization: concepts, measurement, estimation". *Econometrica*, 72(6): 1737--1772. <doi:10.1111/j.1468-0262.2004.00552.x>. The index may be computed for a single or for a range of values of the alpha-parameter and bootstrapping is also available.

**License** GPL (>= 2)

**RoxygenNote** 7.3.3

**NeedsCompilation** yes

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 DER-package

*Income Polarization Index*


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## Description

Description: The PaF income polarization index as proposed by Duclos, Esteban, and Ray. (2004). The index may be computed for a single or for a range of values of the  $\alpha$ -parameter and bootstrapping is also available. In all cases, we first divide the data by  $\mu^{1-\alpha}$ , where  $\mu$  is the mean (income), as described in Duclos, Esteban and Ray (2004). If you want to make the index comparable to the Gini index you should divide the alienation component (and the paf eventually) by 2.

## Details

Package: DER  
 Type: Package  
 Version: 1.5  
 Date: 2026-03-28

## Maintainers

Michail Tsagris <mtsagris@uoc.gr>.

## Author(s)

Michail Tsagris and Christos Adam.

## References

Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

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Bootstrapping the decomposed PaF income polarization index

*Bootstrapping the decomposed PaF income polarization index*

---

## Description

Bootstrapping the decomposed PaF income polarization index

## Usage

```
paf2.boot(y, a, R = 1000, ncores = 1)
```

## Arguments

y	A numeric vector with income data.
a	The value of $\alpha$ . This can be a number only, between 0.25 and 1.
R	The number of bootstrap resamples to perform.
ncores	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

## Details

The function computes the decomposed PaF index of Duclos, Esteban and Ray (2004) for a specific value of  $\alpha$ . The decomposition is with respect to the deprivation and surplus components as suggested by Araar (2008). The PaF index, the deprivation and surplus components, and also their bootstrap estimates, the estimated bias and the estimated standard error of each, and the confidence intervals are returned.

## Value

A list including:

boot	A matrix with the bootstrap estimates.
index	The estimates.
info	A matrix with: the bootstrap based estimates, the bootstrap estimated bias of the estimates, the bootstrap estimated standard errors of the estimates, and the 95% percentile bootstrap confidence intervals for each component.

## Author(s)

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

## References

- Araar A. (2008). On the Decomposition of Polarization Indices: Illustrations with Chinese and Nigerian Household Surveys. CIRPEE Working Paper No. 08-06. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1136](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1136)
- Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

## See Also

[paf2](#), [paf.boot](#)

## Examples

```
y <- rgamma(100, 10, 0.01)
paf2.boot(y, 0.25)
```

---

Bootstrapping the PaF income polarization index

*Bootstrapping the PaF income polarization index*

---

## Description

Bootstrapping the PaF income polarization index

## Usage

```
paf.boot(y, a, R = 1000, ncores = 1)
```

## Arguments

<code>y</code>	A numeric vector with income data.
<code>a</code>	The value of $\alpha$ . This can be a number only, between 0.25 and 1.
<code>R</code>	The number of bootstrap resamples to perform.
<code>ncores</code>	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

## Details

The function compute the PaF index of Duclos, Esteban and Ray (2004) for a specific value of  $\alpha$ , the alienation and identification components, the 1 + normalized covariance, and also their bootstrap estimates, the estimated bias, the estimated standard error of each and the percentile bootstrap confidence interval for the PaF index are returned.

**Value**

A list including:

boot	A matrix with the bootstrap estimates.
index	The estimates.
info	A matrix with: the bootstrap based estimates, the bootstrap estimated bias of the estimates, the bootstrap estimated standard errors of the estimates, and the 95% percentile bootstrap confidence intervals for each component.

**Author(s)**

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

**References**

Duclos J. Y., Esteban, J. and Ray D. (2006). Polarization: concepts, measurement, estimation. In *The Social Economics of Poverty* (pp. 54–102). Routledge.

Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

**See Also**

[paf](#), [paf2.boot](#)

**Examples**

```
y <- rgamma(100, 10, 0.01)
paf.boot(y, 0.25)
```

---

Kernel Density Estimation

*Kernel Density Estimation (KDE)*

---

**Description**

Kernel density estimation of a univariate sample using a Gaussian kernel.

**Usage**

```
kde(y, h, ncores = 1)
```

**Arguments**

y	A numeric vector containing the sample data.
h	A numeric value for bandwidth.
ncores	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

**Details**

The kernel density estimate (KDE) provides a smooth estimate of the probability density function of the sample data. In this implementation, the KDE is evaluated only at the observed sample points themselves.

**Value**

A numeric vector of density estimates corresponding to each observation in y.

**Author(s)**

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

**Examples**

```
set.seed(123)
y <- rnorm(200)
dens <- kde(y, h = 1)
```

---

Many decomposed PaF income polarization indices

*Many decomposed PaF income polarization indices*

---

**Description**

Many decomposed PaF income polarization indices

**Usage**

```
colpafs2(y, a, ncores = 1)
```

### Arguments

y	A numeric matrix with income data. The PaF index will be computed for each column separately.
a	The value of $\alpha$ , a number between 0.25 and 1.
ncores	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

### Details

The function compute the decomposed PaF index of Duclos, Esteban and Ray (2004) for a specific value of  $\alpha$ , for each column of the matrix. The decomposition is with respect to the deprivation and surplus components as suggested by Araar (2008).

### Value

A matrix, where each row contains the PaF index, the deprivation and the surplus components.

### Author(s)

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

### References

Araar A. (2008). On the Decomposition of Polarization Indices: Illustrations with Chinese and Nigerian Household Surveys. CIRPEE Working Paper No. 08-06. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1136](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1136)

Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

### See Also

[paf2](#), [colpafs](#)

### Examples

```
y <- matrix( rgamma(100 * 10, 10, 0.01), ncol = 10 )
colpafs2(y, 0.25)
```

---

Many PaF income polarization indices

*Many PaF income polarization indices*

---

### Description

Many PaF income polarization indices

### Usage

```
colpafs(y, a, ncores = 1)
```

### Arguments

y	A numeric matrix with income data. The PaF index will be computed for each column separately.
a	The value of $\alpha$ , a number between 0.25 and 1.
ncores	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

### Details

The function compute the PaF index of Duclos, Esteban and Ray (2004) for a specific value of  $\alpha$ , for each column of the matrix.

### Value

A matrix, where each row contains the PaF index, the alienation (twice the Gini index) and identification components and 1 + the normalized covariance.

### Author(s)

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

### References

Duclos J. Y., Esteban, J. and Ray D. (2006). Polarization: concepts, measurement, estimation. In *The Social Economics of Poverty* (pp. 54–102). Routledge.

Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.



**See Also**

[paf](#), [colpafs2](#)

**Examples**

```
y <- matrix( rgamma(100 * 10, 10, 0.01), ncol = 10 )
colpafs(y, 0.25)
```

---

The decomposed PaF income polarization index  
*The decomposed PaF income polarization index*

---

**Description**

The decomposed PaF income polarization index

**Usage**

```
paf2(y, a, ncores = 1)
```

**Arguments**

<code>y</code>	A numeric vector with income data.
<code>a</code>	The value of $\alpha$ . This can either be a number or a vector with many values. In any case, the $\alpha$ may take values between 0.25 and 1.
<code>ncores</code>	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

**Details**

The function compute the decomposed PaF index of Duclos, Esteban and Ray (2004) for either a specific value, or for a range of values, of  $\alpha$ . The decomposition is with respect to the deprivation and surplus components as suggested by Araar (2008).

**Value**

For a single value of  $\alpha$ , the function returns a vector with the PaF index, the deprivation and the surplus components. If a range of values of  $\alpha$  are given, it will return a matrix with the same components, where each row corresponds to a specific value of  $\alpha$ .

**Author(s)**

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Christos Adam <econp266@econ.soc.uoc.gr>.

**References**

- Araar A. (2008). On the Decomposition of Polarization Indices: Illustrations with Chinese and Nigerian Household Surveys. CIRPEE Working Paper No. 08-06. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1136](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1136)
- Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

**See Also**

[colpafs2](#), [paf](#)

**Examples**

```
y <- rgamma(100, 10, 0.01)
paf(y, 0.25)
paf2( y, 0.25)
```

---

The PaF income polarization index

*The PaF income polarization index*

---

**Description**

The PaF income polarization index

**Usage**

```
paf(y, a, ncores = 1)
pafF(y, a, ncores = 1)
```

**Arguments**

y	A numeric vector with income data.
a	The value of $\alpha$ . This can either be a number or a vector with many values. In any case, the $\alpha$ may take values between 0.25 and 1.
ncores	The number of cores to use. If greater than 1, parallel computing will take place. It is advisable to use it if you have many observations and or many variables, otherwise it will slow down the process. The default is 1, meaning that code is executed serially.

**Details**

The functions compute the PaF index of Duclos, Esteban and Ray (2004) for either a specific value, or for a range of values, of  $\alpha$ . The `pafF()` estimates the index using Eq. (8) and (9) in the paper, whereas `paf()` is faster as it uses Eq. (3) of the paper.

### Value

The `paf()` function, for a single value of  $\alpha$ , returns a vector with the PaF index, the alienation (twice the Gini index) and identification components and 1 + the normalized covariance. If a range of values of  $\alpha$  are given, it will return a matrix with the same components, where each row corresponds to a specific value of  $\alpha$ .

The `pafF()` function returns only the PaF index for either one or more values of  $\alpha$ .

### Author(s)

Michail Tsagris and Christos Adam.

R implementation and documentation: Michail Tsagris <[mtsagris@uoc.gr](mailto:mtsagris@uoc.gr)> and Christos Adam <[econp266@econ.soc.uoc.gr](mailto:econp266@econ.soc.uoc.gr)>.

### References

Duclos J. Y., Esteban, J. and Ray D. (2006). Polarization: concepts, measurement, estimation. In *The Social Economics of Poverty* (pp. 54–102). Routledge.

Duclos J. Y., Esteban, J. and Ray D. (2004). Polarization: concepts, measurement, estimation. *Econometrica*, 72(6): 1737–1772.

### See Also

[paf.boot](#)

### Examples

```
y <- rgamma(100, 10, 0.01)
paf(y, 0.25)
paf( y, c(0.25, 0.5, 0.75, 1) )
```

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